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Wm Symes Junr 1787

Clavis Arithmeticæ:
Or, A KEY to
ARITHMETICK
William IN Molesworth
Numbers and *Species*;
WHEREIN
ARITHMETICK and ALGEBRA
ARE
Abridged, Demonſtrated and made
Eaſie in their ſeveral *Fundamentals* and *Conſequents.*

Collated from the moſt Authentick Authors on thoſe Subjects, with many Improvements *and Uſeful* Theorems, *not hitherto Extant.*

Begun
By *JOHN PARSONS*, Decd.
And Compleated
By *THO. WASTELL.*

LONDON,
Printed for *Richard Mount* and *Thomas Page*, at the
Poſtern on *Tower-Hill*, 1705.

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THE PREFACE.

Although I am clearly of Opinion with Sir R—— L'E——, *that a Preface to a Book, is only an idle deal of Fiddle Faddle, betwixt the Writer and the Reader, made worſe by Care and Pains; and that in caſe a Book will not ſpeak for it ſelf, all the Author can ſay on its behalf will never recommend it: Yet for the Sa-*

Vide Pref. to his *Tull. Offices.*

tiſfaction of the World, and to do Juſtice to my Deceaſed Friend Mr. Parſons, *I think I ought to give an Account how far I have been concerned in this, and for what Reaſons I meddl'd with it. As to the Firſt, Mr.* Parſons *begun and carried it on to Page* 113, *except the Cuts in Page* 31, 98, 99 *and* 106; *but he Dying, I begun there, and Compleated it as you now find it, and hope I have not only kept to the Text, to make it anſwer its Title, but hit his Deſign in the ſaid Cuts, though I had no Scheme or Directions left for either. And as to the Latter, I was prevailed with by ſome* *of his Relations, and others, of my own Friends, to beſtow a few ſpare Minutes upon it, beſides my own Deſire that ſo Uſeful a Deſign, after ſo much Pains, ſhould not be any longer delayed, Induced me the more chearfully to proceed thereupon, wiſhing it had been Finiſhed by ſome Abler Hand, who might have render'd it more acceptable : Yet I hope the Imperfections will be Excuſed in this Critical and Cenſorious Age, when 'tis Conſider'd I am only a Lover, and not a Profeſſor of the* Mathematicks.

Tho. Waſtell.

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Clavis

Clavis Arithmeticæ.

BOOK I.

CHAP. I.

Notation *of Whole Numbers.*

A*RITHMETICK* is either *Theorical*, that is the *Science* of Numbers; or *Practical*, that is the *Art* of Numbring.

Number is that which anſwers *directly* to the Queſtion, *How many?*

The Fundamental parts of *Arithmetick* are Five; *Viz. Notation*, *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion*.

Notation is that which ſhews how to *expreſs Numbers* by Words and Characters.

> *One, or an Unit is the firſt and leaſt Number, and if to this be put another Unit, there will be another Number; and thus by the continual adding of an Unit there will ariſe an infinite In-*

creaſe and variety of Numbers, for the expreſſing of which were it neceſſary that there ſhould be a like variety of words and characters, Notation *would be an Invincible difficulty: But this is prevented by a Method ſo eaſie to be learnt and remembred, that for its Excellency and Uſefulneſs it may deſervedly be reckon'd amongſt the moſt Admirable Inventions, and yet (for what reaſon I know not, unleſs becauſe it is eaſie and common) it's very rare to find it ſo eſteem'd and obſerved: But if duly conſider'd it may rather ſeem to be a more than Humane Diſcovery, which is the* Encomium *the Learned Dr.* Wallis *hath given of it, in his* Opus Arithmeticum, pag. 22.

The *Notes, Characters* or *Figures* whereby any *Number* is commonly *expreſt*, are only *Ten*, which with their *ſignification* are as followeth.

1	ſignifies	*One*, or an *Unit*	
2		*Two*	*Units*
3		*Three*	
4		*Four*	
5		*Five*	
6		*Six*	
7		*Seven*	
8		*Eight*	
9		*Nine*	
0		*Nothing*, or no *Number* of it ſelf,	

but yet is of great Uſe, as will be ſhewn hereafter. It is called a *Cypher*, and the reſt are called *ſignificant Figures*, or *Digits*, which beſides their ſingle values as above, have alſo other values according to the places wherein they may happen to ſtand.

In every Number there are as many *places* as *Figures*, and the *firſt place* or *Figure* is that next to the right hand, and every ſignificant Figure in the *firſt* place ſignifies only its own ſingle Value, in the *ſecond* place *ten times* as much as in the *firſt*, in the *third ten times* as much as in the *ſecond*, *&c.* continually encreaſing, ſo as that every *Digit* in the *following* place is *ten times* the value of what it would be if it were in the next *aforegoing*.

The Numeration Table.

Place									Period		
9th	8th	7th	6th	5th	4th	3d	2d	1ſt	3d	2d	1ſt
9	8.	7,	6	5	4,	3	2	1	987 *Mil.*	654 *Thou.*	321
	9	8,	7	6	5,	4	3	2	98	765	432
		9,	8	7	6,	5	4	3	9	876	543
			9	8	7,	6	5	4		987	654
				9	8,	7	6	5		98	765
					9,	8	7	6		9	876
						9	8	7	*Nine-hundred eighty ſeven*		
							9	8	*Ninety eight*		
								9	*Nine*		
Hundred-Millions	*Ten-Millions*	*Millions*	*Hundred-Thouſands*	*Ten-Thouſands*	*Thouſands*	*Hundreds*	*Tens*	*Units*	*The Names of Places.*		

It appears by this Table that every 3 *Figures* make a *Period*, and every *Number* is to be *read* beginning at the left hand, and if it conſiſts but of *one* Period or a *part* of a Period, it muſt be *read* in like manner as thoſe 3 Numbers laſt mention'd in the Table: And ſo for the *other Numbers*, having firſt diſtinguiſh't the Periods by Comma's, and as you read, giving every Period its *proper Name*, as the *ſecond* Period that of *Thouſands*, the *third* that of *Millions*. Thus the *Second* Number in the Table, 98, 765, 432 is Ninety eight *Millions*, Seven Hundred Sixty five *Thouſands*, Four Hundred Thirty Two: But by what Method Numbers (how great ſoever they be) may be pronounced with eaſe and readineſs, will beſt be underſtood from what follows.

The *Order* of *Periods.*		The *Order* and *Names* of *Places.*		The *Names* of *Double Periods.*
Double	Single			
3	6	18	*Hundred-Thouſands*	Of *Millions of Millions*, or *Billions.*
		17	*Ten-Thouſands*	
		16	*Thouſands*	
	5	15	*Hundreds*	
		14	*Tens*	
		13	*Units*	
2	4	12	*Hundred-Thouſands*	Of *Millions.*
		11	*Ten-Thouſands*	
		10	*Thouſands*	
	3	9	*Hundreds*	
		8	*Tens*	
		7	*Units*	
1	2	6	*Hundred-Thouſands*	Of *Units*
		5	*Ten-Thouſands*	
		4	*Thouſands*	
	1	3	*Hundreds*	
		2	*Tens*	
		1	*Units*	

Here it is obſervable, that every *ſingle* Period conſiſts of *three* places, and every *double* Period of *ſix.*

The *Names* of the *Double Periods* may be as followeth, *Viz.*

The	1ſt	Double Period is that of	*Units*
	2d		*Millions*
	3d		*Billions* or *Mill.* of *Mill.*
	4th		*Trillions*
	5th		*Quadrillions*
	6th		*Quintillions*
	7th		*Sextillions*
	8th		*Septillions*
	9th		*Octillions*
	&c.		*&c.*

All which being throughly underſtood, it will be eaſie to *read* any Number propoſed in Figures, how many ſoever they be, firſt diſtinguiſhing the *Single* Periods by *Comma's*, and the *double* by *Points*, and then as you read, giving every *double* Period its *proper Name*, or at the Concluſion of of each double Period, repeating the word *Millions* as oft as there are *points* reckoning *that* of this Period with *thoſe* that *follow* towards the right hand.

Example.

1,234,567,892,345,678,934,567,824,321; which is, 1,234 *Quadrillions* or *Millions* of *Mill.* of *Mill.* of *Mill.* 567,892 *Trillions*, or *Mill.* of *Mill.* of *Mill.* 345,678 *Billions*, or *Mill.* of *Mill.* 934,567 *Mill.* 824,321.

A *Cypher* though of it ſelf it hath no value, yet doth it ſerve to fill up a place, and thereby encreaſe the value of other Figures, to which it is prefixt by advancing them a place higher towards the left hand. As for inſtance in the Number 325076, if the Cypher ſhould be wanting, 5 would be in the *third* place, and not in the *fourth*, and therefore would ſignifie Five *hundred*, and not *Five thouſand* and likewiſe the Figures 3 and 2 would each of them be removed a place lower towards the right hand, and conſequently be ten times leſs in value than they are by having a Cypher plac't before them. And thus any *ſingle ſignificant figure*, if it have 1. 2. 3. 4. *&c.* Cyphers ſet before it will thereby be advanced to the place of, (and therefore ſignifie ſo many) *Tens*, *Hundreds*, *Thouſands*, *Ten-Thouſands*, *&c.* as there are *Units* in the ſignificant Figure.

COROL. I.

Hence it plainly follows that the prefixing of 1. 2. 3. &c. *Cyphers to any whole Number doth make it Ten, an Hundred, a Thouſand times greater*, &c. Examp. *If before the whole Number* 235 *be placed* 1.2.3 *Cyphers*, &c. *every figure or part thereof will be* 1. 2. 3. *places*, &c. *higher towards the left hand, whereby all the parts, and conſequently the whole will be Ten, an Hundred, a Thouſand times greater*, &c.

COROL. II.

So that the prefixing of the ſame Number of Cyphers to any whole Numbers will equally increaſe the values of thoſe Numbers.

Notation of *Numbers by Letters.*

1	I	90	XC.
2	II	100	C.
3	III	400	CCCC.
4	IIII. IV.	500	D. Iↄ.
5	V.	600	DC. IↄC.
6	VI.	1000	M. CIↄ.
7	VII.	4000	MMMM.
8	VIII.	5000	Iↄↄ.
9	IX.	6000	IↄↄC.
10	X.	10,000	CCIↄↄ.
11	XI.	50,000	Iↄↄↄ.
15	XV.	100,000	CCCIↄↄↄ.
19	XIX.	1,000,000	CCCCIↄↄↄↄ.
20	XX.	10,000,000	CCCCCIↄↄↄↄↄ.
30	XXX.		
40	XL.		
50	L.	1694.	M DC XCIIII.
70	LXX.		

CHAP. II.

Addition *of Whole Numbers.*

ADDITION is that which ſhews how to bring ſeveral Numbers into *one*, which is called the *Total*; or when *ſeveral* Numbers are *given*; how to find out a Number which ſhall be *equal* to all of them together.

RULE.

RULE.

Firſt ſo order the Numbers given that the Figures of like degree or place may be directly under each other, that is, Units under Units, Tens under Tens, &c. Then add them together, beginning with the rank of units, and ſet the ſum of each rank, if it be leſs than Ten, under its reſpective rank, but if it amount to Ten, or any number of Tens, with ſome Over-plus, ſubſcribe that Over-plus, but if without any, ſet down a Cypher, and always for every Ten reſerve One or an Unit to be added to the next rank towards the left hand.

Example.

Let the ſeveral Sums in the Margin be given to be added together, which being order'd as before directed, begin with the Rank of Units, ſaying,

9526
9984
8791
———
28301 *Total.*

1 & 4	make	5	
5 & 6		11	ſet down 1 & carry 1
1 carried & 9		10	
10 & 8		18	
18 & 2		20	ſet down 0 & carry 2
2 carried & 7		9	
9 & 9		18	
18 & 5		23	ſet down 3 & carry 2
2 carried & 8		10	
10 & 9		19	
19 & 9		28	ſet down 28

The **Reaſon** of this Operation is evident from the Nature of *Notation*, according to which every *Ten* of the Rank *aforegoing* makes one of the next *following*.

The Proof of **Addition.**

Work again, beginning at the *top* of each Rank inſtead of the *bottom*, and if the Total be the *ſame*, you may ſafely conclude that the Operation is *true*.

CHAP. III.

Subſtraction *of Whole Numbers.*

SUBSTRACTION is the *taking* one Number *from* another, ſo as to find out the *Remainder* or *Difference*.

RULE.

Firſt place the leſſer under the greater in ſuch order as is directed in Addition, and then beginning at the right hand, ſubſtract each figure below from that above, and ſet down the ſeveral Remainders underneath, and the Number ſubſcribed will be the Total difference or remainder of the two Numbers.

Example.

Let 3514 be given to be Subſtracted from 9758; having plac't them as in the Margin;

$$\begin{array}{rl}9758 & \\ 3514 & \\ \hline 6244 & \text{the } \textit{difference} \\ \hline & \text{or } \textit{remainder}\end{array}$$

$$\text{Say}\left\{\begin{matrix}4\\1\\5\\3\end{matrix}\right\}\text{from}\left\{\begin{matrix}8\\5\\7\\9\end{matrix}\right\}\text{and there remains}\left\{\begin{matrix}4\\4\\2\\6\end{matrix}\right\}$$

which ſet down underneath and the Number *ſubſcribed* will be 6244 the *Remainder* or *Difference* of the Two Numbers.

The Operation is in effect thus.

$$\left.\begin{array}{r}4\\10\\500\\3000\\\hline 4514\end{array}\right\}\text{from}\left\{\begin{array}{r}8\\50\\700\\9000\\\hline 9758\end{array}\right\}\begin{array}{c}\text{and there}\\\text{remains}\end{array}\left\{\begin{array}{r}4\\40\\200\\6000\\\hline 6244\end{array}\right.$$

The **Reaſon** of this Operation is manifeſt fro that known *Axiom, the whole is equal to all its part* ſo that the ſubſtracting of all the *parts* is equivalent to the Subſtraction of the *whole Number* given to be ſubſtracted.

Note; *That ten (that is 1 borrowed from the nex place to the left) is to be added to th Figure above, as oft as it is leſs tha that below it; and the* 1 *is to b repaid by adding it to the Figure whic is next to be ſubſtracted.*

Exampl

Example.

Let 608 be given to be ſubſtracted from 945 which being plac't in due order, ſay 8 from 15 (that is 10 and 5) and there remains 7, which ſet down, 1 (that is, 1 borrowed, and 0 is 1) from 4 and there remains 3, which ſet down, 6 from 9, and there remanis 3, which ſet down, and the Number *ſubſcribed* will be the *remainder.*

$$\begin{array}{rl} 945 & \\ 608 & \\ \hline 337 & \text{Remainder} \end{array}$$

The **Reaſon** hereof is that, by *borrowing* 1 from the next Figure following, it is ſuppoſed to be *diminiſh't* 1, ſo that the Figure below having 1 *added* to it, thereby 1 more is *ſubſtracted* from that above, and the *remainder* the ſame, as if that above were really *leſſen'd* 1.

Subſtraction may be perform'd by *beginning* at the *left* hand. As if 597 be given to be *ſubſtracted* from 753, then *beginning* with 5, ſay 5 from 7, and there *remains* 2 (that is 500 from 700 and the *Remainder* is 200) which 2 *ſet* above, and *daſh* out the 5 and 7, then ſay 9 from 15 and there *remains* 6 (that is 90 from 150,) and the *Remainder* is 60, which 6 ſet above, and *daſh* out the 5 and 9, and 1 (borrowed) from 2 there *remains* 1 (that is 100 from 200, and the *remainder* is 100) which 1 *ſet* above and daſh out 2, *Laſtly,* 7 from 13, and there *remains* 6, which *ſet* above, and *daſh* out 7 and 3, and 1 borrowed from 6, there *remains* 5 (that is 10 from 60, and the *remainder* is 50) *ſet* the 5 above, and

$$\begin{array}{r} 15 \\ \cancel{2}\cancel{6}6 \\ \cancel{7}\cancel{5}\cancel{3} \\ \cancel{5}\cancel{9}\cancel{7} \\ \hline 156 \end{array}$$

and *daſh* out 6, and the Figures 1 5 6 which are *not daſht* make the *remainder*. This Method of *Subſtraction* is very tedious, but becauſe there is a like Method uſed in *Diviſion* (as ſhall be ſhewn hereafter) that the *reaſon* thereof might be the better underſtood, I thought it convenient ſo *particularly* to explain the foregoing Operation.

The Proof of Subſtraction.

Add the *Remainder* to the *leſſer* Number, and if the *Total* be the ſame with the *greater*, the work is certainly *true*. For the *Remainder* is nothing elſe but the *defect*, or what the leſſer *wants* to make it *equal* to the greater, and therefore both added together muſt make up the greater.

CHAP. IV.

Multiplication *of Whole Numbers.*

MULTIPLICATION is the taking a Number given *ſo many times, or parts of a time, or ſo much of a time or times* as ſhall be deſir'd, thereby to find out what it will *produce*, or it is the finding out a Number which ſhall *contain* another given *ſo oft* as ſhall be required.

The Number given to be multiplied is called the *Multiplicand*, that by which it is to be multiplied is the *Multiplier*, and the Number ſought or which ariſes by multiplying the one by the other is called the *Product*. *Mul-*

Multiplication is either *ſingle*, that is, when the *Multiplicand* and *Multiplier* do each conſiſt of *one* Figure only, or *Compound*, that is when *either* or *both* conſiſt of *more* Figures than *one*.

Single Multiplication is performed in the following Table.

The Table of Multiplication.

2 . 2	4	3 . 7	21	5 . 8	40
2 . 3	6	3 . 8	24	5 . 9	45
2 . 4	8	3 . 9	27	6 . 6	36
2 . 5	10	4 . 4	16	6 . 7	42
2 . 6	12	4 . 5	20	6 . 8	48
2 . 7	14	4 . 6	24	6 . 9	54
2 . 8	16	4 . 7	28	7 . 7	49
2 . 9	18	4 . 8	32	7 . 8	56
3 . 3	9	4 . 9	36	7 . 9	63
3 . 4	12	5 . 5	25	8 . 8	64
3 . 5	15	5 . 6	30	8 . 9	72
3 . 6	18	5 . 7	35	9 . 9	81

In this Table are 3 *double* Columns, in each of which, againſt every *two ſingle* Numbers ſtands their reſpective *Product*, as, in the third Column right againſt 6 and 9 (that is 6 times 9) is plac't 54 (the *product* thereof.)

Note, *That 9 times* 8, *or* 8 *times* 9 *make the ſame Product, and ſo for any other Numbers, as* 36 *times* 4 *is the ſame with* 4 *times* 36, *which (if requir'd) may be proved thus.*

36 times { 1, 1, 1, 1 } is the ſame with *once* { 36, 36, 36, 36 } which plainly proves that 36 *times* 4 is the ſame with 4 *times* 36, and the *reaſon* is the ſame for all other Numbers. This *Table* muſt be ſo fixt in the Memory, that one may be able readily to give the *Product* of any two *ſingle* Figures, which will make it much more eaſie to work *Compound Multiplication*, as followeth.

RULE I.

In Compound Multiplication, if the Multiplier conſiſts of one Figure only, ſet it under the firſt Figure of the Multiplicand, and having drawn a line underneath, let each Figure thereof, beginning with the place of Units, be multiplyed by the Multiplyer, and ſet each ſingle Product (if leſs than ten) under its reſpective Figure of the Multiplicand, but if it be ten or any Number of tens with ſome over-plus, ſubſcribe that over-plus, but if without, ſet down a Cypher, and always for every ten reſerve one to be added to the next product, and the number ſubſcribed will be the product of the whole.

Example.

Let 7306 be given to be multiplied by 6, which being plac't as in the Margin, ſay,

7306 *Multiplicand*
6 *Multiplier*

43836 *Product*

6 times	6	make	36	set down	6 & carry 3.
	0		0 & 3 carried make 3.		3.
	3		18		8 & carry 1.
	7		42 & 1 carried make 43.		43.

The Reaſon of this Rule is ſufficiently evident from the Nature of *Notation*, according to which every *ten* in the *aforegoing* ſingle Product makes *one* of the next *following*.

RULE 2.

When the Multiplier doth conſiſt of more than one ſignificant figure, let the Multiplicand be multiplyed by each figure of the Multiplier as before directed, beginning with the firſt, and placing the ſeveral Products thereof underneath each other in ſuch order, that the firſt Figure or Cypher of each Product may be in the ſame place with its reſpective multiplying Figure; then add theſe particular Products together, and the Total thereof will be the product of the whole Multiplication.

Example.

Let the	*Multiplicand*	be	5232
	Multiplier		4032

Then will 5232 multiplyed by	2	be	10464	which *Products* added together, will
	3		15696.	
	4		20928...	

be ——— ——— - ——— 21095424, and this is the *total product* of the whole *Multiplication*.

The

The Reaſon *of this* Operation.

The *firſt multiplying Figure* (2) being in the place of *units*, it is apparent (by the firſt *Rule* hereof,) that its reſpective *Product* muſt be ſo placed, as that the *firſt figure* thereof may be alſo in the place of *units*. The ſecond *figure* (3) in the Multiplier being in the place of *tens*, and therefore of *ten times* greater value than if it were in the *units* place, its *product* muſt alſo be *ten times* greater, and conſequently to make it ſo, the *firſt figure* thereof muſt be in the place of *tens*, as appeares by the Nature of *Notation*, and by the ſame reaſon (4) being in the place of *Thouſands* in the Multiplier, the firſt *figure* of its *product* muſt be advanced to the *like place*: From whence it is evident that the *firſt* figure of each *particular* Product muſt be in the *ſame* place with its reſpective multiplying figure; And the *Multiplicand* being multiplied by all the *parts* of the Multiplier, it is manifeſt that the *Sum* of thoſe Products will be the *Product* of the *whole*, which is *equal to all its parts.*

RULE 3.

If either or both the Numbers given begin with one or more Cyphers, cut off the Cyphers, and multiply the remaining figures by each other, and to the Product prefix all the Cyphers.

Example.

Let the	*Multiplicand*	be	7950
	Multiplier		3700

Then will 795 multiplyed by 37 *produce* 29415, to which if the *Cyphers* be added, the *whole Product* will be 29415000, as appears by the following Operation.

```
 795|0
  37|00
 --------
 5565
2385
 --------
 29415000  Product.
```

Note, *That if the Multiplier be* 1, *with one or more Cyphers before it, the Operation may be performed only by prefixing the Cyphers to the Multiplicand,* (*per* Corol. 2. Chap. 2.) *Thus,*

The *Product* of					
	73	*multipl. by*	10	is	730
	65		100		6500
	720		1000		720,000
	300		10000		3,000,000

The Reaſon *of this Rule.*

If the Multiplier have 1. 2. 3 Cyphers, &c. annext, it will be 10, 100, 1000 *times* greater, &c. and conſequently the Product muſt be *as many times* greater, *that is*, ſo many Cyphers muſt be added to it as are in the Multiplier (*per Corol.* 2. *Chap.* 1.) and by the ſame reaſon, if the Multiplicand have 1. 2. or more Cyphers annext, it will likewiſe be 10, 100 *times* greater, &c. and therefore the Product muſt again be made *ſo many times* greater, by adding *more* Cyphers to it, that is, *ſo many*, as (*per Corol.* 2. *Chap.* 1.) are required to make it *ſo much* greater: From whence it appears that there muſt be *as many* Cyphers prefixt to the *Product* as there are to *both* the Numbers given.

CHAP. V.

Diviſion *of Whole Numbers.*

DIVISION is the taking *ſuch a part* of a Number given, or the dividing it into *as many equal parts*, as ſhall be deſir'd, or the finding out *how oft* one Number is *contained* in another.

The Number given to be divided is called the *Dividend*, and that by which it is to be divided is the *Diviſor*, and the Number ſought is called the *Quotient*. Thus if 72 be given to be divided into 9 equal parts, or by 9, then 72 will be the Dividend, and 9 the Diviſor, and that which anſwers to the Queſtion, *What is the* 9th *part of* 72, *or how oft* 9 *is contained in* 72? is the Quotient.

RULE I.

If the Diviſor be a ſingle Figure, take the firſt Figure of the Dividend towards the left hand, if it be not leſs than the Diviſor, *but*, if it be, *take it and the next for a Dividual, and enquire how oft the Diviſor is contain'd therein, and ſet down the anſwer in the Quotient, by which multiplying the Diviſor, ſubſtract the Product from this Dividual, and let* *the Remainder (if any) withthe next Figure or this Figure (if no remainder) be a new dividual, and having found how oft the Diviſor is contain'd therein, place the anſwer in the Quotient, and multiply and ſubſtract as before directed, but if any Dividual after the firſt happen to be leſs than the Diviſor, ſet a Cypher in the Quotient, and take this and the next for a new Dividual, and ſo continue the Operation until all be finiſhed.*

Example.

Diviſor	*Dividend*	*Quotient*
4)	176032	(44008
	16 ···	
	16	
	16	
	00032	
	32	
	00	

Let 176032 be propoſed to be *divided* by 4. Firſt, Ask how oft 4 in 17, the anſwer is 4, which ſet in the Quotient, and by it having multiplyed the Diviſor (4) the Product will be 16, which being ſubſtracted from the Dividual (17) there will remain 1, and this with the next Figure (6) makes a new Dividual (16) in which the Diviſor (4) being contain'd 4 times, place 4 in the Quotient, and then the Product of 4 multiplyed by 4 ſubſtracted from this dividual, there will remain nothing, and the next Figure being a Cypher, ſet a Cypher in the Quotient, then take the next Figure (3) which being leſs than the Diviſor, ſet another Cypher in the Quotient, and let this and the next Figure (2) that is 32, be a new Dividual, in which the Diviſor (4) being contained exactly 8 times, place 8 in the Quotient, and the remainder will be nothing.

Such Operations may eaſily be performed in the mind, thus in the following Example, by *enquiring*

How oft 5 in	anſwer,	
32	6	
29	5	
40	8	times
0	0	
5	1	time

Dividend
Diviſor 5) 329005
Quotient 65801

R U L E 2.

365	1
730	2
1095	3
1460	4
1825	5
2190	6
2555	7
2920	8
3285	9

If the Diviſor conſiſt of more Figures than one; Firſt make a Table to ſhew the Product of the Diviſor taken any Number of times under ten according to the example in the Margin. The Numbers on the right ſide of the line ſhew how oft the Diviſor is taken, and thoſe on the left are the ſeveral Products thereof, Thus againſt 1 *is once* 365, *and againſt* 2 *twice* 365, &c. *and if to this laſt product be added* 365, *there will be thrice* 365, *and thus you may proceed until the Table be finiſht, and if to the laſt Number againſt* 9 *you add the firſt, and it make* 365 *with a Cypher, that is, ten times* 365, *you may then conclude the Table is true, this laſt Number being deriv'd from thoſe aforegoing. Having thus prepar'd a Table of the Diviſor, diſtinguiſh for a Dividual by a point above or below, ſo many Figures of the Dividend towards the left hand as are in the Diviſor (if they do not make a number leſs than it,) but, if they do, then take one Figure more for the Dividual, and ſee for a Number in the Table that is either equal or next leſs to it, and this Tabular Number (after the Figure ſtanding againſt it is plac't in the Quotient)*

tient) you muſt ſet under and ſubſtract from this Dividual, and to the Remainder (if any) prefix the next Figure in the Dividend towards the right hand, having firſt ſet a point under or above it, and there will be a new Dividual, then ſeek for a Tabular Number as before directed, and proceed as you did with the firſt Dividual, and ſo on until the work be ended, but if, after you have brought down a Figure, the Number ſubſcribed happen to be leſs than the Diviſor, then ſet a Cypher in the Quotient.

Example.

```
365) 1096825 (3005
     ....
     1095
     ----
        1825
        1825
        ----
        ....
```

Let 1096825 be given to be divided by 365, of which having made a Table as before, and pointed out the firſt Dividual, which is 1096, (for 109 is leſs than 365,) ſeek for this Number in the Table, or the next leſs to it, and in this caſe you will find it to be 1095, which (after the figure ſtanding againſt it is plac't in the Quotient) muſt be ſubſtracted from the Dividual, and there will remain 1, then putting a point under the next Figure (8) bring down and prefix it to 1, and there will be 18, which being leſs than the Diviſor, place a Cypher in the Quotient, and having pointed the next Figure (2) prefix that likewiſe, and the Number ſubſcribed will be 182, which being likewiſe leſs than the Diviſor, put another Cypher in the Quotient and prefix the next Figure 5, and this new Dividual will be 1825, which ſtanding in the Table againſt 5, put 5 in the Quotient, and the Tabular Number being equal to the Dividual, there will remain nothing, and ſo the Operation is finiſhed.

This *Method of Diviſion* in uſing a Table of the Diviſor is of *great uſe* in *large* Diviſions, or when there is a conſtant or frequent occaſion for the *ſame* Diviſor, but in other caſes it being too tedious, I ſhall in the next place ſhew how to *divide* a Number *without* the help of a Table.

RULE 3.

Try how oft the firſt Figure of the Diviſor towards the left hand is contained in that of the Dividual, if there be as many Figures in that as in this; *But,* if there be not, *then try how oft in the* two *firſt Figures, yet always ſo that the remainder of the Diviſor may be found as oft in that of the Dividual, and the figure thus found being plac't in the Quotient by it, multiply the Diviſor, and proceed as before directed, and ſo for every Dividual until the Operation be concluded.*

Examp. Let 93571 be given to be divided by 137. Firſt of all enquire how oft the firſt figure (1) of the Diviſor (137) is in the firſt figure (9) of the firſt Dividual (935). Now though there

may be ⎫ 9 times ⎧ 1, the *1ſt.* ⎫ fig. of the Diviſor (137) in ⎧ 9, the *1ſt.* ⎫ fig. of the Dividual (935)
yet not ⎭ ⎩ 3, the *2d.* ⎭ ⎩ 3, the *2d.* ⎭

Therefore try 7 times, ſaying,

7 times
1 is 7 this from 9, reſts 2, this with 2^{d} fig. 3 is 23.
3 is 21 this from 23, reſts 2, this with 3^{d} fig. 5 is 25.
7 is 49 which cannot be ſubſtracted from 25.

```
    22
137)935
    7..
    21.
    49
```

wherefore make another Tryal as followeth.

6 times
1 is 6 this from 9, reſts 3, which with 2d fig. 3 is 33.
3 is 18 this from 33, reſts 15 wch with 3d fig. 5 is 155.
7 is 42 this from 155 reſt 113.

```
    (11
    35(5
137)935
     6
    18
    42
```

This laſt Tryal ſhews how oft the Diviſor is contain'd in the *firſt Dividual*, and the like Method of making Tryals is to be uſed in the *other Dividuals*.

Thus the *Diviſor* is found to be in each *Dividual* as followeth, *viz.*

in the 1*ſt*, 6 / 2*d*, 8 / 3*d*, 3 times and therefore 6 / 8 / 3 to be placed in the *Quotient*.

And the reſt of the Operation is thus,

Diviſor Dividend Quotient
137) 93571 (683

935 The firſt *Dividual*.
Firſt 6 *times* 137 makes —— 822 which ſubſtracted,
there remains 113, which with the next fig. 7 makes 1137 the ſecond *Dividual*.
Then 8 *times* 137 makes —— 1096 which ſubſtracted,
there remains 41 which with the next fig. 1 makes 411 the third *Dividual*.
Laſtly 3 *times* 137 makes —— 411 which ſubſtracted,
there remains nothing —— and the work is *finiſhed*.

This *Method of Diviſion* may be a little alter'd only by *placing* the remainder *above* the Dividend, and *Cancelling* the Figures as you operate, Thus,

$$\begin{array}{lrrl}
 & & \cancel{4} & \\
 & & \cancel{1}\cancel{1}\cancel{3}\cancel{1}(1 & \\
 & 137) & \cancel{9}\cancel{3}\cancel{5}\cancel{7}\cancel{2} & (683 \\
 & & 822\cdot\cdot & \\
 & & 1096\cdot & \\
\text{The } \textit{Total} \text{ of the} & & 411 & \\
\textit{Diviſors} \text{ products} & \left.\begin{array}{c}\\ \\ \\ \end{array}\right\} & \overline{} & \\
\text{with the Remaind.}(1) & & 93572 &
\end{array}$$

By comparing *this* Operation with the *next foregoing*, it is eaſy to perceive that it is in effect the *ſame*, with *this advantage*, that here the ſeveral Products of the Diviſor ſtand more conveniently to be *added together*, which with the remainder (if any) muſt alwayes be *equal* to the Dividend, otherwiſe the work is *falſe*.

RULE 4.

Firſt place the Diviſor under the firſt Dividual, and having as before diſcover'd the firſt Figure of the Quotient, by it let each Figure in the Diviſor, (beginning at the left or right hand, and cancelling each Figure as you operate) be ſeverally multiplyed, and each Product ſubſtracted from the Dividual, and the remainder ſet above, then let the Diviſor be removed a place forwarder towards the right hand, and having found the next Figure of the Quotient, multiply and ſubſtract as before, and thus proceed until the Operation be concluded.

Example.

Let 7532 be given to be divided by 217. Firſt having plac't the Diviſor 217 under the firſt Dividual, and found the firſt figure in the Quotient to be 3, begin with the firſt figure in the Diviſor (2) next the left hand, and ſay

3 times 2 make 6, 6 from 7 and there reſts 1, which ſet over 7 having cancell'd the 2 and 7.—

$$\begin{array}{l} 1 \\ \cancel{7}532\,(3 \\ \cancel{2}17 \end{array}$$

3 times 1 make 3, 3 from 5 and there reſts 2, which ſet over 5 having cancell'd the 1 and 5.—

$$\begin{array}{l} 12 \\ \cancel{7}\cancel{5}32\,(3 \\ \cancel{2}\cancel{1}7 \end{array}$$

3 times 7 make 21, 21 from 23 (that is 2 being borrowed from the next place) and there remains 2 which ſet over 3, and cancel 3 and 7, then 2 (borrowed) from 2 and there reſts 0, which ſet over and cancel 2.———

$$\begin{array}{l} \;\,0 \\ 1\cancel{2}2 \\ \cancel{7}\cancel{5}\cancel{3}2\,(3 \\ \cancel{2}\cancel{1}\cancel{7} \end{array}$$

Then Remove the Diviſor a place forwarder towards the right hand and find out how oft the Diviſor is in the new Dividual (1022) which are the uncancell'd figures, as above, and the 2*d.* figure in the Quotient will be 4, then ſay,

4 times 2 make 8, 8 from 10 and there remains 2, which ſet over 0, having cancell'd the 2 and 10.———

$$\begin{array}{l} \;\,2 \\ \;\,\cancel{0} \\ \cancel{1}\cancel{2}2 \\ \cancel{7}\cancel{5}\cancel{3}2\,(34 \\ \cancel{2}\cancel{1}\cancel{7}7 \\ \;\,\cancel{2}1 \end{array}$$

4 times

4 times 1 make 4, 4 from 12, and there remains 8, which ſet over 2, having cancell'd the 1 and 2, then 1 (borrowed) from 2, reſts 1, which ſet over, and cancel 2.————	1 2 08 122 7532(34 2177 21
4 times 7 make 28, 28 from 32 and there remains 4, which ſet over 2, having cancell'd the 2 and 7, then 3 (borrowed) from 8 there reſts 5, which ſet over and cancel 8.————	(1 2(5 08 122(4 7532($34\frac{154}{217}$ 2177 21

Laſtly, To *conclude* the Operation, *place* 154 (the uncancell'd Figures) above 217 (the Diviſor) with a line between, and this will *ſignifie* 154 of the *parts* of an Unit divided into 217 *equal parts*, or (which is all one) the 217th part of 154, as ſhall be hereafter explain'd in treating of the *Nature of Fractions.*

Note, *That the like is to be done for all remainders after Diviſion, if it be requir'd to give the exact Quotient.*

The other way, which differs from the former only, beginning at the right inſtead of the left hand of the Quotient, is as followeth.

Having plac't the Diviſor, and found the firſt figure in the Quotient as before, ſay,

3 times 7 make 21, this from 23
there remains 2, which ſet over
3, 3 times 1 make 3, and 2
(borrowed) make 5, this from
5 reſts 0, which ſet over 5;
3 times 2 make 6, this from }
there rem. 1, which ſet over } 7

$$\begin{array}{l} 102 \\ \cancel{7}\cancel{5}\cancel{3}2(3 \\ \cancel{2}\cancel{1}\cancel{7} \end{array}$$

Then having removed the Diviſor, and found the ſecond figure in the Quotient, ſay,

4 times 7 make 28, this from 32
there remains 4, which ſet over 2
4 times 1 make 4 and 3 (borrow'd) make 7, this from 12,
reſts 5, which ſet over 2, 4 times
2 make 8, and 1 (borrow'd)
make 9, this from 10, reſts 1,
which ſet over 0, and ſo the Diviſion is ended, and there remains
154 as before.

$$\begin{array}{l} (15 \\ \cancel{1}\cancel{0}\cancel{2}(4 \\ \cancel{7}\cancel{5}\cancel{3}2(34\tfrac{154}{217} \\ \cancel{2}\cancel{1}\cancel{7}\cancel{7} \\ \;\cancel{2}\cancel{1} \end{array}$$

This Method of Operation may be a little varied, only by placing the *remainders*, inſtead of the *Diviſor*, underneath the Dividend, as followeth,

$$\begin{array}{ccc} \textit{Diviſor} & \textit{Dividend} & \textit{Quotient} \\ 217) & 7532 & (34\tfrac{154}{217} \\ & \;\;\dot{}\,\dot{} & \\ & 102 & \\ & \;\;154 & \end{array}$$

And this compar'd with the former will eaſily be underſtood to be in effect the ſame Operation.

Note, *That when the Diviſor doth begin with one or more Cyphers, they muſt be cut off from the Diviſor, and as many of the firſt figures towards the right hand from the Dividend, and then the Diviſion to be perform'd with the remaining Figures according to any of the foregoing Rules, and when the Operation is finiſhed, the remainder (if any) with the figures cut off is to be placed above, and the whole Diviſor below, as in the following Example.*

137|00)94943|32(693$\frac{232}{13700}$
127
41

The laſt Remainder —— 2 with 32 cut off, will be 232 to be placed over the whole Diviſor, as above.

The **Reaſon** of the *foregoing* Rules may ſufficiently appear by conſidering the *following* inſtance.

Diviſor *Dividend*
708) 4112772 (5809
····
1ſt *Divid.* 4112··· is *really* 4112000 } { 5000
3540···
—— ··
2d *Divid.* 5727·· is *really* 572700 } and therefore its Quotient is { 800
5664··
—— ··
3d *Divid.* 6372 is —— 6372 } { 9
6372
—— The *whole Quotient* 5809
····

☞ In the aforegoing Operation it is to be obſerv'd that by finding how oft the Diviſor is contain'd in the ſeveral *parts* of the *Dividend* thereby is diſcover'd how oft it is contain'd in the *whole*. Thus the firſt *Dividual* (4112) *part* of the Dividend (4112772) contains the Diviſor 5 *times* with an *Overplus* of 572, but 4112 being *really* 4112000 (becauſe there are three places before it in the Dividend) and therefore 1000 *times greater* than 4112 conſider'd by it ſelf, the *Quotient* muſt alſo be 1000 *times* greater, that is 5000 and the *overplus* 572 with the next Figure 7 in the Dividend makes 5727 the ſecond *Dividual*, which being *really* 572700 (becauſe there are two places before it in the Dividend) will contain the Quotient 800 *times* with an *overplus* of 63, which with the next Figure 7 makes *really* 6370, but not containing the Diviſor ſo oft as *once* with reſpect to the next place, that is, not *once ten times*, therefore a *cypher* muſt be put in the place of tens in the *Quotient*, and the next Figure (2) in the Dividend being prefixt to 637 will make the third *Dividual really* 6372, in which the Diviſor is contain'd *really* 9 *times* without any remainder, and the *whole Quotient* is exactly 5809. *Q. E. D.*

In the inſtance of Dividing 9494332 by 13700 it is evident that this Diviſor by having the 2 Cyphers cut off is made 100 *times leſs*, and therefore the Dividend muſt be made *as much leſs*, which is done by cutting off *as many places* from it as are cut off from the Diviſor, otherwiſe the Quotient would be 100 *times greater* than it *really* ſhould be, for 137 being 100 *times leſs* than 13700 is contain'd in the *whole Dividend* (9494332) 100 *times* oftner than the *whole Diviſor* is contain'd therein.

CHAP. VI.

Of the Proof of Multiplication *and* Diviſion.

MULTIPLICATION and DIVISION prove each other. For *what is done by Multiplication is undone by Diviſion, and what is undone by Diviſion is done by Multiplication.* Now by the nature of Multiplication the *Multiplier* ſhews how oft the *Multiplicand* is *taken*, or *contain'd* in the *Product*, and if this Product be divided by the Multiplicand, the *Quotient* will alſo ſhew the *ſame*, and conſequently be equal to the Multiplier, if the Product be *true*. And ſince either of the two Numbers given may be the *Multiplicand*, and the other the *Multiplier*, and in both caſes the *Product* will be the *ſame*, therefore this Product may be divided by either, and the Quotient will be the other.

Likewiſe in *Diviſion* the *Quotient* ſhewing how oft the *Diviſor* is contain'd in the *Dividend*, therefore if the Diviſor be taken ſo many times as there are Units in the Quotient (*that is*, if the Diviſor be multiplied by the Quotient, or, *which is all one*, the Quotient by the Diviſor) it will make the *Product* equal to the *Dividend.*

Example

Example. 1.

Operation	Proof	
73 *Multiplicand*	73) 4088 (56	56) 4088 (73
56 *Multiplier*	365	392
438	438	168
365	438	168
4088 *Product*	...	...

Example 2.

Operation and Proof.

Diviſor Dividend Quotient
5) 535 (107 multiplied
by 5

535 equal to the *Dividend*

Example 3.

Operation *Diviſor Dividend Quotient*	Proof
29) 347 (11	29 *Multiplicand*
29	11 *Multiplier*
57	29
29	29
28 the *Remainder*	28 the *Remaind.*
the *Product*	347 equal to the *Dividend*

In *Examp.* 1. If the Operation be *true*, the *Multiplier* (56) ſhews how oft the *Multiplicand* (73) is contain'd in the *Product* (4088) that is, 56 *times*, for 56 *times* 73 is equal to 4088, and ſince the *Quotient* of 4088 divided by 73 ſhews *likewiſe* how oft 73 is contain'd in 4088, therefore this *Quotient* muſt *alſo* be 56, *that is*, it muſt be equal to the *Multiplier*: Or 73, if it be the *Multiplier*, ſhews how oft the *Multiplicand* (56) is contained in the ſame *Product* (4088) and if this 4088 be divided by 56, the *Quotient* will alſo ſhew the ſame, *that is*, it will be equal to 73 the *Multiplier*.

And in *Examp.* 2, Since the *Quotient* 107 ſhews how oft the *Diviſor* (5) is contain'd in the *Dividend* (535) therefore 107 *times* 5 (*that is*) 5 multiplied by 107, or *which is all one*, 107 by 5 will make up the *Dividend*, or a *Product* equal to the *Dividend* (535).

Laſtly, In *Examp.* 3. It appears that 347 (the *Dividend*) contains 29 (the *Diviſor*) 11 *times*, and 28 over, ſo that 11 *times* 29 (or 29 multiplied by 11) and 28 added, will make up 347 the *Dividend.*

There is the *ſame reaſon* for that other *way* of examining *Diviſion*, *viz.* by adding the *particular* Products of the *Diviſor* (as they ſtand in the Operation) together with the *remainder* (if any) the *Total* whereof will be equal to the *Dividend*, if the work be *true*.

Example.

Diviſor Dividend Quotient
283) 154519 (546
1415 . . — 1*ſt* particular *Product*

1301
1132 — 2*d.* ——————
1699
1698 — 3*d.* ——————
and . . . . . 1 the *Remainder* ——
} added togeth.

154519 the *Total* equal to the *Dividend.*

Or,

$\cancel{1}\cancel{6}$
$\cancel{1}\cancel{3}\cancel{0}\cancel{9}$(1 *Remainder*
283) $\cancel{1}\cancel{5}\cancel{4}\cancel{5}\cancel{1}\cancel{9}$ (546
1415 . .
1132 .
1698
} particular *Products* added together.

make 154519 the *Total* as before.

Here the particular *Products* are added together without *examining* whether they *truely* ariſe from the *Diviſor* multiplied by each Figure in the *Quotient*, ſo that, if either there happen to be a Figure *omitted*, or a *falſe* one put in the *Quotient* (which may be, though the Diviſor be multiplied by the *true* one which is carried in the mind) this way of its ſelf will not *diſcover* ſuch errors without conſidering *further* how many *places* there ought to be in the *Quotient*, and alſo trying one or two of the *firſt Figures* in each *Product*, whether or no they will agree with *as many* in the *Diviſor* multiplyed by each reſpective Figure in the *Quotient*.

There is *another way* of proving *Multiplication* and *Diviſion*, which is both *ſhorter* and *eaſier*, but not ſo *certain* as the *former*, nor yet ſo *erroneous* (as is commonly imagin'd) that it ſhould be *wholly* rejected.

It is thus, in *Multiplication* having made a Croſs, add all the Figures in the *Multiplicand*, as if they were ſo many Units, and caſting away the Nines, ſet the *Remainder* on *one* ſide of the Croſs, and doing the like in the Multiplier, ſet this *Remainder* on the *other* ſide, and ſo in the Product ſetting its *Remainder* on the top; laſtly multiply the figures (on the ſides of the Croſs) by each other, and having caſt away the Nines, if *this Remainder* be equal to *that* on the top, it may well be preſum'd that the work is *true*, but *if not*, that it is *falſe*. The like *Method* is to be uſed in proving *Diviſion*, by ſuppoſing either the *Diviſor* or *Quotient* to be the *Multiplicand*, and the other the *Multiplier*, and the *Dividend* to be the *Product*.

Example 1.

Multiplicand 647
Multiplier 86

3882
5176

Product 55642

$$\begin{array}{ccc} & 4 & \\ 8 & \times & 5 \\ & 4 & \end{array}$$

I.

$$\text{The fig.}\left\{\begin{array}{c}7\\4\\6\end{array}\right\}\begin{array}{l}\text{of the}\\\textit{Multiplicand}\\\text{added together}\end{array}$$

make —— 17 wherein
is once —— 9 which deducted

Reſts —— 8 to be ſet on *one* ſide of the *Croſs.*

II.

$$\text{The fig.}\left\{\begin{array}{c}6\\8\end{array}\right\}\begin{array}{l}\text{of the}\\\textit{Multiplier}\\\text{added together}\end{array}$$

make —— 14 wherein
is once —— 9 which deducted

Reſts —— 5 to be ſet on the *other* ſide of the *Croſs.*

III.

4
8 X 5
4

$$\text{The fig.}\left\{\begin{array}{c}2\\4\\6\\5\\5\end{array}\right\}\begin{array}{l}\text{of the}\\\textit{Product}\\\text{added together}\end{array}$$

make —— 22 wherein is twice 9,
that is, —— 18 which deducted

Reſts 4 to be placed on the *top* of the *Croſs.*

Laſtly, The Figures, 8 and 5, on the *ſides* of the *Croſs*, multiplyed by each other, make 40 in which is 4 times 9, *that is*, 36 which deducted,
Reſts —— 4 equal to 4 on the top of the *Croſs*.

Or the *Nines* may be caſt away as followeth.

Example 2.

Multiplicand 5232
Multiplier 4032

10464
15696
20928

Product 21095424

0
3 X 0
0

I.

The fig. {2 3 2 5} of the *Multiplicand* added together

make 12

Then {1 2} added together

make 3 to be placed on *one ſide* of the *Croſs*.

2. The

II.

The fig. $\left\{\begin{matrix}2\\3\\0\\4\end{matrix}\right\}$ of the *Multiplier* added together

make —— 9 which being caſt away, there remains 0 to be placed on the *other ſide* of the *Croſs*.

III.

The fig. $\left\{\begin{matrix}4\\2\\4\\5\\9\\0\\1\\2\end{matrix}\right\}$ of the *Product* added together.

make —— 27

Then $\left\{\begin{matrix}7\\2\end{matrix}\right\}$ added together

make —— 9 which caſt away, there remains 0 to be placed on the *top* of the *Croſs*.

Laſtly, The Figures 3, 0, on *each ſide* the *Croſs* multiplyed by each other make 0, which is the *ſame* with *that* on the *top* of the *Croſs*, and therefore the Operation may ſafely enough be concluded to be true.

If in *Diviſion* there be any *Remainder*, let it be added to the *Product* of the Figures on each ſide of the *Croſs*, and then caſt away the *Nines* as before.

Example.

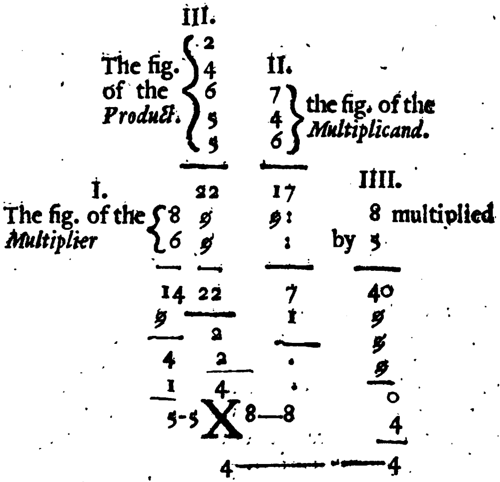
7
8
16) 387 (24 3
32 ———
——— 18
67 ———
64 8
——— 1
3 the *Remain.* ———
6 9 4
1 ——— 2
——— 0 ———
7—7 X 6—6
0

7 multiplied
by 6
———
makes 42
———
2
4
3 the *Remainder*
———
9 to be caſt away
———
Remains 0

The Foundation of *this Method* of proving *Multiplication* and *Diviſion*, is the *reckoning* for every 10 in the place *foregoing* 1 to be put in the place next *following*. For to put 1 for 10 is the ſame thing, as to caſt away, or ſubſtract 9 from 10; or to put 2 for 11 (that is 1 for the 10, and 1 for the 1) as to caſt away 9 from 11. This will farther appear by conſidering the *following* Operation.

Multiplicand 647
Multiplier 86
42
$\cancel{9}$ ·
$\cancel{9}$ ·
$\cancel{9}\cancel{9}\cancel{9}$ ·
$\cancel{9}\cancel{9}\cancel{9}$ ·
$\cancel{9}24$ ·
36 · ·
3882 1*ſt. particular Product.*
: 56 ·
· $\cancel{9}$ ·
· $\cancel{9}$ ·
$\cancel{9}\cancel{9}\cancel{9}$ ·
$\cancel{9}\cancel{9}\cancel{9}$ ·
$\cancel{9}\cancel{9}\cancel{9}$ ·
$\cancel{9}32$ ·
48 · ·
5176 2*d. particular Product.*
$\cancel{9}$
The Total *Prod.* 55642
$\cancel{9}\cancel{9}$

3. The


III.
The fig. of the *Product.*
2 4 6 5 5
II.
7 4 6
the fig. of the *Multiplicand.*
I.
The fig. of the *Multiplier*
8 6
22
14 22
4
1
5-5
2
2
4
17
7
1
8—8
IIII.
8 multiplied by 5
40
0
4
4—4

Here all the *Nines* that are in effect *caſt away* by putting 1 for every 10, are thoſe which are *daſht*, as in the beginning of the Operation 6 *times* 7 make 42, where 2 is ſet down for the 2, and 4 for the 40, *that is*, 4 *times* 9 are *caſt away*, and ſo for the other *ſingle Products*, and in adding the ſingle and particular *Products* together, and carrying 1 for every 10 there are more *Nines caſt away*, ſo that by *caſting away* the *Nines* in the *Total Product*, there are all *caſt away* that are contain'd in all the *ſingle Products* of each ſingle figure in the *Multiplicand* multiplied by each figure in the *Multiplier*, conſider'd without any reſpect to their places, as if they were ſo many Units. Now ſince, *Equal Quantities multiplied by Equal Quantities will make Equal Products.*

And the fig.	6,4,&7	of the	*Multiplic.*	being equal to	9&8
	8,&6		*Multiplier*		9&5

Therefore 6, 4 and 7 multiplyed by 8 and 6, will be equal to 9 and 8 multiplyed by 9 and 5, ſo that each of theſe *Products* will have the ſame number of *Nines*, and the *ſame* Overplus (if any), and conſequently after all the *Nines* in each product are *caſt away*, the Remainders will be equal. And ſince any Number multiplyed by one or more *Nines* will make a certain number of *Nines*, *therefore* it will be needleſs to multiply 9 and 8 by 9, or 9 by 5, becauſe the *Nines* are to be *caſt away*; *wherefore* there will remain only 8 and 5 to be multiplied by each other, and the *Nines* to be caſt out of the *Product* to find whether *this* remainder be equal to the *other* as it ought to be if the *Operation* be performed *aright.*

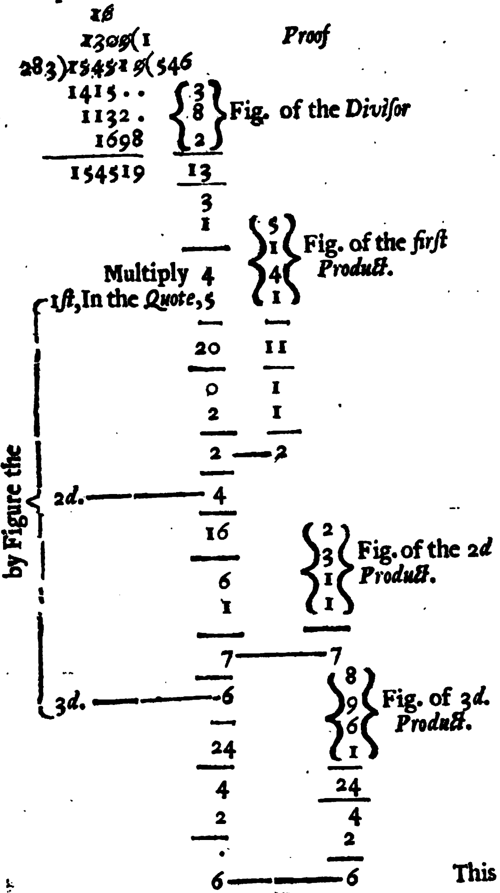
It is to be confeſt that *this Method* will not diſcover an Error made by *miſplacing* a figure, becauſe herein all Figures are conſider'd without any reſpect to their places, but it will certainly detect all Errors that ariſe by ſetting down only *one falſe Figure* in the Operation, or *two*, unleſs the one ſhall happen to be as much *too little* as the other ſhall be *too big*; or indeed by putting *falſe figures* how many ſoever they may be, unleſs for thoſe which (being added together) are *leſs*, there be others that (being alſo added together) are as much *bigger*, than the ſum of the true ones, but that there ſhould at any time be ſuch a *combination* of Errors, except it be *deſignedly*, ſeems more than *improbable.*

The reaſon of ſo *fully* explaining this *laſt Method* of proving Multiplication, is for the *ſatisfaction* of ſuch as ſhall prefer it to any other way, and alſo becauſe it may juſtly be preſum'd that the true *ground* and *reaſon* thereof is generally but *very little* underſtood by thoſe who are the moſt *forward* to condemn it as *utterly* uſeleſs.

Moreover I ſhall here ſhew how *advantagiouſly* it may be uſed in *large* Diviſions, and that is by proving every *particular Product* of the Diviſor as you operate, which will *preſently* diſcover an error in any *ſingle figure* of the Quotient before you proceed to the *next*.

Example.

Operation *Example.*



This *proof* may be eaſily perform'd in the *mind*, having firſt caſt away the *Nines* out of the Diviſor, and ſet down the *Remainder*, which is always to be *multiplyed* by each *Remainder* of each particular *Product*.

The *like Method* may be uſed for the *proof* of *Multiplication*, by examining each *particular Product* before you proceed to the next.

CHAP. VII.

The Nature *and* Notation *of* Decimal Fractions.

AN *Unit* may be ſuppoſed to be *divided* into, or to *contain* 10 *equal* parts, and each of theſe likewiſe into 10 *equal* parts, and then the *Unit* will be ſuppoſed to *contain* 10 *times* 10 (*that is*) 100 *equal* parts, and thus by a continual *ſubdiviſion* into 10 *equal* parts, an *Unit* may be conceived to be *divided* into 10, 100, 1000 *equal* parts, *&c. infinitely*; which are called 10th, 100th, 1000th *parts* of an *Unit*, &c.

According to *this Diviſion* it is apparent that an *Unit* doth contain 10 *tenths*, 1 *tenth* 10 *hundredths*, &c.

Theſe parts are called *Decimal Fractions*, and their places are *below* that of Units according to the following Table.

A Table of Notation *of* Decimals.

	&c.		&c.	
8	10 *Millionths*	Or the parts of an *Unit* divided into	10000000	equal parts
7	*Millionths*		1000000	
6	100 *Thouſandths*		100000	
5	10 *Thouſandths*		10000	
4	*Thouſandths*		1000	
3	*Hundredths*		100	
2	*Tenths*		10	
1	*Units*			
2	*Tens*			
3	*Hundreds*			
4	*Thouſands*			
5	10 *Thouſands*			
6	100 *Thouſands*			
7	*Millions*			
8	10 *Millions*			
	&c.			

In this Table it is to be *obſerv'd* that *as* the Figures in the *firſt*, *ſecond*, *third* places, *&c.* *above* that of Units ſignifie *ſo* many *Tens*, *Hundreds*, *Thouſands*, &c. So the like *Figures* in the *firſt*, *ſecond*, *third* places, *&c.* *below* that of Units ſignifie *as* many *Tenth*, *Hundredth*, *Thouſandth* parts of an *Unit*, *&c.* And *as* in *whole* Numbers, *ſo* in *Decimals* every place to the *left* hand is *ten* times the value of that next the *right*, an *Unit* containing 10 *Tenths*, 1 *Tenth* 10 *Hundredths*, &c.

A *Decimal Fraction* (alone or with a whole Number) is *diſtinguiſh't* by prefixing a point to the Decimal, as in the Examples *following*.

.596 3.7809 40.724.

A *Decimal* is to be *read* as a whole Number giving it the name of the *laſt* place thereof to the *right* hand. Thus,

.3	ſignifie	3 *Tenths*
.75		75 *Hundredths*
.575		575 *Thouſandths*
.3968		3968 *Ten-Thouſandths.*

Cyphers before *Decimal Fractions* removing them ſo many places towards the *right* hand do *as* much *diminiſh* the value thereof *as* the like number of *Cyphers* before *whole Numbers*, removing them the like Number of places towards the *left* do *encreaſe* the values of theſe Numbers, that is 1, 2, 3, &c. Cyphers, removing the Decimal figures as many places lower than before do make them 10, 100, 1000 *times leſs*, &c.

Example.

.39	ſignifie	39 *Hundredths*
.039 . . .		39 *Thouſandths*
.0039 . .		39 *Ten-Thouſandths*
.00039 .		39 *Hundred-Thouſandths*
.309 . . .		309 *Thouſandths*
.3009 . .		3009 *Ten-Thouſandths*
.30009 .		30009 *Hundred-Thouſandths*

COROL.

From what hath been ſaid, it is eaſie to obſerve, That if a Number have 1, 2, 3 *Decimal* places, *&c*, it will be 10, 100, 1000 times *leſs*, *&c.* than if it were a whole Number. Thus if in the *whole* number 375 you make *one Decimal* place, as 37.5, it will be 10 *times leſs*, and if you make 2, 3, 4 *Decimal* places therein, as 3.75, .375, .0375, *&c.* it will be *leſſened* 100, 1000, 10000 *times*, *&c.* For when 5 is removed into the firſt *Decimal* place, it is then 10 *times leſs* than ſo many Units, and 7 being then in the place of Units is *ten times leſs* than 70, and ſo 3 being removed a place lower out of the *hundreds* into the *tens* place is likewiſe *ten times leſs*, and conſequently all the *parts* being *ten times leſs*, the *whole* will be *ſo*. And thus it appears that every removal of a figure into a place towards the *left* doth make it *ten times leſs* than it was before, and therefore to *leſſen* a Number 10, 100 *times*, *&c.* there muſt be 1, 2 *Decimal* places, *&c.* therein, and that is done by prefixing a point to *ſo* many *figures* as there are *Decimal* places required, and if there muſt be more *Decimal* places than there are *Figures* in the number, then ſo many *Cyphers* muſt be prefixt as will make up the *Decimal* places required. As for inſtance, to *leſſen* 375, 10000 times will require four *Decimal* places, which make 1 more than there are *Figures* in 365, and therefore *one Cypher* muſt be plac't before it as .0375, which is 10000 *times leſs* than the *whole Number* 375.

CHAP.

CHAP. VIII.

Addition *and* **Subſtraction** *of* **Decimals.**

RULE.

ADDITION and SUBSTRACTION of Decimals *are the ſame as in* whole Numbers; *always obſerving to place every figure of the ſame degree, or place under the like place above, and imagining all void places to be ſupplyed with Cyphers.*

Examp. of **Addition.**

79.63	76.45	.1	
24.75	43.796	.905	
89.53	7.0395	.7968	
193.91	127.2855	1.8018	Total.

Examp. of **Subſtraction.**

352.09	.576	79.	
63.74	.0829	.2987	
288.35	.4931	78.7013	Remain.

CHAP. IX.

Multiplication of Decimals.

RULE.

THE Multiplication *of* Decimals *is likewiſe the ſame with that of* whole Numbers, *onely in the Product there muſt alwayes be as many* Decimal *places as there are in both the Multiplier and Multiplicand.*

Examp.

Multiplicand 759.2
Multiplier 5.037

53144
22776
37960

Product 3824.0904

Multiplicand 0096
Multiplier 072

192
672

Product 0006912

.45
.029

405
90

.01305

Demonſtration.

The *reaſon* of this Operation will be manifeſt by conſidering the *Corol.* in the *Notation* of Decimals. As for inſtance in the ſecond Example hereof, the *Multiplicand* .0096 being 10,000 *times leſs* than if it were a *whole Number* 96, the *Product* muſt alſo be 10,000 *times leſs*, and therefore have as many *Decimal* places as are in the *Multiplicand* to make it *ſo*, and becauſe the *Multiplier* .072 is 1000 *times leſs* than if it were a *whole Number* the *Product* muſt be *leſſen'd* 1000 *times* more, and therefore have ſo many *more Decimal* places as are in the *Multiplier*. So that there muſt be as many *Decimal* places in the *Product* as are in both the *Multiplier* and *Multiplicand*. W. W. D.

CHAP. X.

Diviſion of Decimals.

RULE 1.

THIS *Operation is alſo the ſame with that of* Diviſion *in* whole Numbers, *but in finding out the true value of the Quotient, it is to be* obſerv'd, *That the* Diviſor *being plac't under the firſt Dividual, the figure anſwering it in the Quotient, muſt always be in a like place with that figure in the Dividend which is over the Units place of the Diviſor.*

Example.

Let .0006528 be propoſed to be *divided* by .032 which being placed (or ſuppoſed to be placed) under the firſt Dividual .00065 it appears *thus*, that the ſecond *Decimal place* in the *Dividend* ſtands over the *place* of *Units* in the *Diviſor*, and therefore the firſt *Figure* (2) ariſing in the *Quotient* muſt alſo be in the ſecond *Decimal place*, ſo that the *firſt* place is to be ſupply'd with a *Cypher*, and the whole *Operation will be as followeth.*

.0006528
.032

.032) .0006528 (.0204
64
128
128
...

Here is to be *obſerv'd* that the whole *Operation* is performed *without* the Cyphers prefixt to the Decimals.

Note, *That if either the* Dividend *have not ſignificant Cyphers ſufficient for thoſe of the* Diviſor, *or if after the Diviſion there be a* remainder, *by* annexing Cyphers, *you may proceed to what degree of exactneſs you pleaſe, as in the* following *Examples.*

```
53) 1.70000 (.03207
      159···
      ───
      110
      106
      ────
        400
        371
       ────
         29

27) 123.5000 (4.57407
       ·  ····
    108
    ───
      155
      135
     ────
       200
       189
      ────
        110
        108
       ────
         200
         189
         ───
          11
```

Demonſtration.

The **Reaſon** of the aforegoing Rule for finding out the *value* of the *Quotient* may eaſily be underſtood by obſerving, that *as* in *whole Numbers*, if the firſt place (towards the left hand) of the *firſt Dividual* be in the place of Tens, Hundreds, *&c.* above that of Units, the *Quotient* will be 10, 100 *times greater*, &c. than if the ſaid firſt place were in that of Units: *So in Decimals* if the firſt place of the *firſt Dividual* be in that of 10ths, 100ths, *&c.* below that of Units (the *Dividual* being then 10, 100 *times leſs*, &c. than if the firſt place thereof were in that of Units) the figure anſwering it in the *Quotient* will likewiſe be 10, 100 *times leſs*, &c. That is, it will be of the *ſame value* with the *laſt* Decimal place in the *Dividual*, *that is*, that place which is over the place of Units (that is the firſt place) in the *Diviſor*, when it is a *whole Number*: But if the place of Units, in the *Diviſor*, be ſuppoſed to be removed to the ſecond, third place, *&c.* as every removal thereof to a place towards the *left hand* doth bring it under a place in the Dividual of ten times *greater* value than that next the *right*, *ſo* by *diminiſhing* the Diviſor 10 *times*, it doth likewiſe make the figure anſwering it in the Quotient of 10 *times greater* value than before, that is, of the *ſame* value with that place in the *Dividual* which is over the place of Units in the *Diviſor*.

As ſuppoſe the *Dividend* be .0006528, and the *Diviſor* 32, it is manifeſt that the *firſt* figure 5 of the *Dividual* (.00065) being in the 5*th. Decimal* place, the *Dividual* is 100,000 *times leſs* than if it were a *whole Number*, and therefore the figure anſwering it in the *Quotient* muſt be as many *times leſs*, and conſequently ſet in the like Decimal place with the figure 5 in the *Dividual* which ſtands over 2 in the Units place of the *Diviſor*, as it is a whole

.00065
32
.00065
3.2
.00065
.32
.00065
.032

Number. But *as* the *place* of Units in the *Diviſor* by being advanced a *place* higher to the *left* will come under the fourth place in the *Dividual* which is of 10 *times greater* value than the aforegoing place, *ſo* in this caſe, the *Diviſor* being made 10 *times leſs*, the Figure anſwering it in the *Quotient*, muſt be 10 *times greater* than before, that is, of the *ſame* value with 6 which is now over the place of Units in the *Diviſor*, and the *reaſon* is evidently the ſame for *every removal* of the Units place in the *Diviſor* towards the *left*: So that the *firſt figure* ariſing in the *Quotient* muſt alwayes be of the *ſame* value with that place which at the *firſt* Queſtion, is over that of *Units* in the *Diviſor*. W. W. D.

RULE 2.

There is another Rule for finding out the value of the Quotient, *viz; After the Diviſion is ended, conſider how many Decimal places there are in the Dividend, for ſo many there muſt be in the Quotient. as the Dividend hath more than the Diviſor.*

The **Reaſon** of this Rule is evident from the Nature of *Multiplication*, the *Dividend* being in effect the *Product* of the *Diviſor* and *Quotient* multiplyed together, ſo that what *Decimal* places are *more* in the Dividend than the Diviſor, muſt be ſupplied in the *Quotient*, ſince according to the Rule of *Multiplication* in *Decimals*, they muſt both together have as many as are in the *Dividend*.

CHAP. XI.

The Nature and Notation *of* Vulgar Fractions.

AN *Unit* or *Integer* may be conſider'd as a *whole* diviſible not only into 10, 100, &c. as in *Decimals*, but into any *other* Number of equal parts which are called *Broken Numbers* or *vulgar Fractions*, and are uſually *expreſt* by two Numbers, the one *above* the other, with a line between them, *thus* $\frac{1}{5}$ or $\frac{3}{4}$, that *below* is called the *Denominator*, becauſe it *denominates* or *ſhews* what parts the Unit is ſuppoſed to be *divided* into, and the other *above* is called the *Numerator*, becauſe it *expreſſeth* the Number of theſe parts, or how many of them are *ſignified* by the Fraction. As $\frac{1}{5}$ *ſignifies* 1 fifth part of an Unit, and $\frac{3}{4}$ *ſignifies* 3 fourth parts of an Unit, or if an Unit be *divided* into 4 equal parts, this Fraction *ſignifies* 3 of thoſe parts.

There is alſo *another Notion* of a *Fraction*. For ſince not only one *Unit* or *Integer*, but any *Number* of Integers (that is as many as are expreſt in the *Numerator*) may be conſider'd as a *whole* diviſible into any number of equal parts (that is as many as are expreſt by the *Denominator*). Therefore a Fraction may *ſignifie* one of theſe parts, and then $\frac{3}{4}$ will *ſignifie* 1 fourth of 3, or if 3 be *divided* into four equal parts, this Fraction is 1 of theſe parts. This *latter Notion* of a *Fraction* is in effect the ſame with the *former*: As for inſtance $\frac{3}{4}$, that is (according to the *former Notion*)

3 *fourth* parts of 1 Unit is the ſame Quantity with $\frac{3}{4}$ taken (according to the *latter Notion*) for 1 *fourth* of 3 Units. For,

$\frac{3}{4}$, or $\left\{\begin{array}{l}\text{1 \textit{fourth} of 3 Units}\\ \text{3 \textit{fourths} of 1 Unit}\end{array}\right\}$ is thrice 1 *fourth* of 1 Unit, or $\frac{3}{4}$.

That Fraction, whoſe *Numerator* is *leſs* than its *Denominator*, is called a *proper* Fraction; but if its *Numerator* be *greater* than, or *equal* to its *Denominator*, it is then called an *Improper* Fraction.

The *Numerator* of a Fraction is *ſo* many times *greater* than the Fraction, *as* there are Units in the *Denominator*. Thus 3 being ſuppoſed to be divided into 5 *equal* parts is 5 times *greater* than one of theſe parts, or the Fraction $\frac{3}{5}$.

The *greater* or *leſſer* the *Numerator* of a Fraction is, the *greater* or *leſſer* is the Fraction. As $\frac{3}{5}$, *that is*, 3 *fifths* of an Unit is 3 *times* greater than 1 *fifth* of the ſame Unit, and thus $\frac{3}{9}$ *that is*, 3 *ninths* of an Unit is but half as much as $\frac{6}{9}$ *that is*, 6 *ninths* of the ſame Unit, or which is all one, 1 *ninth* of 3 is but half as much as 1 *ninth* of 6.

The *greater* or *leſſer* the *Denominator* of a Fraction is, on the *contrary*, the *leſſer* or *greater* will the Fraction be. For the *more* or *fewer* the parts are into which the whole is divided, the *leſſer* or *greater* will one of thoſe parts be. As $\frac{1}{5}$ is 3 *times greater* than $\frac{1}{15}$, or $\frac{4}{5}$ than $\frac{4}{15}$. For when the whole is ſuppoſed to be divided into, (or to contain) 5 equal parts, and each of theſe parts to be ſubdivided into, (or to contain) 3 equal parts, then will the whole be divided into 3 times 5, (*that is*) 15 equal parts, and conſequently 1 of the *fifth* parts, containing 3 of the *fifteenth* parts will be 3 *times greater* than 1 of theſe *fifteenth* parts, and the reaſon is the ſame for other Fractions.

COROL.

COROL. 1.

A fraction is	*multiplied* by *multiplying*	its *Numerator*
	divided by *dividing*	
	multiplied by *dividing*	its *Denominator*
	divided by *multiplying*	

For the Fraction whoſe *Numerator* is *multiplied*, is made ſo many times *greater* as there are *Units* in the *Multiplier*, but if the *Denominator* be *multiplyed*, it is made ſo many times *leſs*: And if the *Numerator* be *divided*, the Fraction is made ſo many times *leſs* as there are *Units* in the *Diviſor*, but if the *Denominator* be *divided*, it is made ſo many times *greater*.

COROL. 2.

If both termes of a Fraction be *divided* by the ſame Number, the Quotients will make another Fraction of the ſame value: For then what the Fraction *loſes* of its value by the Dividing its *Numerator* it *regains* by dividing its *Denominator*: And on the contrary, if the terms of a Fraction be multiplyed by the ſame Number, the Products will be the terms of another Fraction of equal value. For then as the Fraction, by multiplying its *Numerator*, will be *encreaſed*: So will it, by multiplying its *Denominator*, be as much *Diminiſhed*, that is, it will be reduced to the ſame value as at firſt.

If what hath been here diſcourſt concerning the nature of Fractions be throughly underſtood, the reaſons of the following Operations will appear very obvious, and diſcover the whole Miſtery thereof, and ſo make them much more eaſie to be remembred. *Note*, That the Signs + — × = ⊐ (uſed in the following Chapters) are explained in *chap.* 1. *lib.* 2.

CHAP. XII.

The Reduction *of* Vulgar Fractions.

PROP. I.

To find the greateſt common meaſure of two given Numbers.

RULE.

Divide the *greater* by the *leſſer*, and this *Diviſor* by the *Remainder* (if any) and ſo on continually, till there be *no* remainder left; and the *laſt* Diviſor will be the *greateſt* common meaſure, *that is*, the *greateſt* Number that will *divide* both the Numbers given without *any* Remainder.

Example. 1.

Let the Numbers given be 252 and 364.

252) 364 (1
252
———
112) 252 (2
224
———
The greateſt common meaſure is —— } 28) 112 (4
112
———
...

28) 252 (9
———
28) 364 (13
28
———
84
84
———

Exampl.

Example 2.

Let the given Numbers be 17 and 153.

The greateſt common Meaſure or *Diviſor* is } 17) 153 (9 17) 17 (1

153

———

...

17)153(9

Example. 3.

Let the Numbers given be 289 and 841.

2)(1 (8 (10 (1 (2 Quotients

1)2) 3) 26) 263) 289) 841

2 2 24 260 263 578

0 1 2 3 26 263 { Remainders and Diviſors

1 Being their *greateſt* common Meaſure they are called *Primes*.

Demonſtration.

The *leſſer* of the two Numbers is the greateſt *common Meaſure* when it will *exactly* divide the greater, but when it will *not*, the greateſt *common Meaſure* is the greateſt *aliquot* or *even* part common to both, one of which doth exactly contain a *greater*, and the other a *leſſer* number of *ſuch* parts, and after the *greater* is divided by the *leſſer*, the *aliquot* parts in the *remainder* will be *fewer* than thoſe in the *Diviſor*, and ſo continuing the Operation according to the Rule, by the ſame reaſon, every *Diviſor* and *Dividend* conſiſting of theſe parts, and every Diviſor *following* being *leſs* than the *foregoing* the Number of the *aliquot* parts remaining after every Diviſion muſt be continually *leſſen'd* until at laſt there remain but *one* of *them*, and this will exactly divide the next foregoing Diviſor, and conſequently the *laſt Diviſor* which leaves no remainder will be the greateſt *common meaſure*, or Number that will divide both the Numbers *given* without any remainder. *W. W. D.*

But to *illuſtrate* what hath been ſaid, put C for the greateſt *common meaſure*, and let a certain Number of C^s^ be put for the *greater* Number given, and a *leſſer* Number of C^s^ for the *leſſer* Number given, *Viz.*

CCCCCCC) CCCCCCĊCCCCCCĊCC (2
CC) CĊCĊCĊC (3
c) cċ (2 exactly without any remainder. All which duely conſider'd is a *plain demonſtration* of the aforegoing Rule.

PROP. II.

To Reduce a Fraction into it's leaſt termes.

RULE.

Divide the *Numerator* and *Denominator* by their *greateſt* common meaſure, and their *Quotients* will be the *Numerator* and *Denominator* of a Fraction *equal* to the former, and in the *leaſt terms.*

Example 1.

Let the Fraction given be $\frac{252}{364}$

The *greateſt common meaſure* 28) 252 (9
found *per Prop.* 1. 28) 364 (13

Example 2.

Let the Fraction be $\frac{17}{153}$

The *greateſt common* 17) 17 = (1
meaſure 17) 153 (9

Demonſtration.

The dividing *both* terms of the Fraction by the *ſame* Number doth produce *another* Fraction of the *ſame* value, (*per Corol.* 2. *Chap.* 9.) and the Diviſor being the *greateſt* that will divide both, the Fraction will alſo be in the *leaſt* termes. For the *greateſt* Diviſor makes the *leaſt* Quotient.

If the *greateſt* common meaſure be 1, the Fraction is *already* in the *leaſt* termes, ſince the dividing them by 1 will *not* diminiſh them.

Beſides the *aforegoing* general Rule there are *other* uſeful Rules as followeth.

1. *When the Termes are even, divide them by 2 continually.*

Example.

The Fraction $\frac{48}{272}$ will be $\frac{3}{17}$ in the *leaſt* terms, as appears by the following Operation.

$$\begin{array}{r|c|c|c|c} \begin{matrix}2)\\2)\end{matrix}\frac{48}{272} & \frac{24}{136} & \frac{12}{68} & \frac{6}{34} & \frac{3}{17} \end{array}$$

2. *When the termes do both begin with 5, or one with 5, and the other with a Cypher, divide them by 5 continually.*

Examp.

$\frac{175}{435}$ will be *reduced* to $\frac{7}{13}$ and $\frac{75}{400}$ to $\frac{3}{17}$, as followeth.

$$\begin{array}{r|c|c} \begin{matrix}5)\\5)\end{matrix}\frac{175}{425} & \frac{35}{85} & \frac{7}{17} \end{array} \qquad \begin{array}{r|c|c} \begin{matrix}5)\\5)\end{matrix}\frac{75}{400} & \frac{15}{80} & \frac{3}{16} \end{array}$$

3. *Divide the termes by the greateſt Numbers that may be readily diſcover'd will exactly divide them.*

Example.

$\frac{42}{126}$ is *reduced* to $\frac{6}{18}$ by dividing the terms by 7, and $\frac{6}{18}$ to $\frac{1}{3}$, by 6, as follows.

$$\frac{42}{126} \Bigg| \frac{6}{18} \Bigg| \frac{1}{3}$$

4. *When both termes begin with Cyphers, cut off equal Cyphers in both, and if the Fraction be not then in the leaſt termes, proceed to operate according to the foregoing Rules.*

Examp.

$\frac{300}{500}$ will be $\frac{3}{5}$ and $\frac{90}{4500}$ will be $\frac{1}{90}$ as follows.

$$\frac{3}{5} \Bigg| \frac{00}{00} \qquad \frac{9}{450} \Bigg| \frac{0}{0} \Bigg| \frac{1}{90}$$

The Reaſon of theſe Operations is manifeſt from *Corol.* 2. *Chap.* 9.

PROP. III.

To reduce a Compound Fraction into a ſingle one.

RULE.

Multiply all the *Numerators* together for a *new* Numerator, and all the *Denominators* together for a *new* Denominator.

Example.

Let the *Compound* Fraction given be $\frac{3}{4}$ of $\frac{7}{6}$ of $\frac{4}{5}$

Multiply	3	and	4
by	7	by	6
Multiply	21	and	24
by	4	by	5
The laſt *Products*	84		120

So that $\frac{3}{4}$ of $\frac{7}{6}$ of $\frac{4}{5}$ is $\frac{84}{120} = \frac{7}{10}$ in the *leaſt* terms.

Demonſtration.

Take *firſt* $\frac{7}{6}$ of $\frac{4}{5}$. Now it is manifeſt that $\frac{1}{6}$ of $\frac{4}{5}$ is $\frac{4}{5}$ *divided* by 6, which (by *Cor.* 1. *ch.* 11.) will be $\frac{4}{5\times6}$ and becauſe $\frac{7}{6}$ is 7 *times* $\frac{1}{6}$, therefore $\frac{7}{6}$ of $\frac{4}{5}$ will be 7 *times* $\frac{4}{5\times6}$ (*that is*, by the ſame Cor. $\frac{4\times7}{5\times6}$) and by the *ſame* reaſon $\frac{3}{4}$ of $\frac{4\times7}{5\times6} = \frac{4\times7\times3}{5\times6\times4}$ *Ergo* $\frac{3}{4}$ of $\frac{7}{6}$ of $\frac{4}{5} = \frac{4\times7\times3}{5\times6\times4}$ $\left(=\frac{84}{120}=\frac{7}{10}.\right)$ *W. W. D.*

PROP. IV.

To reduce a mixt Number into an Improper Fraction.

RULE.

Multiply the *Integers* or *whole Number*, and the *Denominator* of the Fraction by each other, and to the *Product* add the *Numerator* and the *Total* will be a *new* Numerator, and the Denominator the *ſame* as before.

Examp.

Example.

Let the mixt Number given be $395\frac{11}{16}$

$$\begin{array}{lr} \text{Multiply} & 395 \\ \text{by} & 16 \\ \hline & 2370 \\ & 395 \\ & 11 \\ \hline \text{The } \textit{Prod.} \text{ with } 11 \text{ added, is} & 6331 \end{array}$$

$$\frac{6331}{16} = 395\frac{11}{16}$$

The Reaſon of this Rule.

In this inſtance the Unit being ſuppoſed to be divided into 16 *equal parts*, there muſt be *as many times* 16 *as* there are Units in the whole Number, that is, the whole Number muſt be *multiplied* by 16, and the *Product* will be 16^{ths}, to which muſt be added the 11 *ſixteenths*, and then the *Total* will be *Sixteenths* alſo.

PROP. V.

To Reduce a whole Number into the Form of an Improper Fraction.

RULE.

Make the *whole* Number *given* the *Numerator*, and an *Unit* its *Denominator*.

Example.

Let the whole Number given be 225, and its equivalent Improper Fraction will be $\frac{225}{1} = 225$ The *reaſon* of this is ſelf-evident.

PROP. VI.

To reduce a whole Number into an Improper Fraction which ſhall have a Denominator aſſign'd.

RULE.

Multiply the *whole Number* given, and the *Denominator aſſign'd* by each other, and to the Product *ſubſcribe* the ſaid Denominator.

Example.

Let the whole Number given be 63, and the Denominator *aſſign'd* 23,

$$\begin{array}{rr} \text{Multiply} & 63 \\ \text{by} & 23 \\ \hline & 189 \\ & 126 \\ \hline \textit{Product} & 1449 \end{array}$$

$$\frac{1449}{23} = 63$$

The Reaſon of this Rule is the ſame with that in *Prop.* 4.

PROP.

PROP. VII.

To reduce an Improper Fraction into its equivalent whole or mixt number.

RULE.

Divide the *Numerator* by the *Denominator*, and the *Quotient* will be the *whole* or *mixt* Number.

Examp, I.

Let the given Fractions be $\frac{1541}{23}$ and $\frac{6331}{16}$.

$$\begin{array}{lcl}
23)\,1541\,(67 = \frac{1541}{23} & \quad & 16)6331(395\frac{11}{16} = \frac{6331}{16} \\
\underline{138\cdot} & & \underline{48\cdot\cdot} \\
161 & & 153 \\
\underline{161} & & \underline{144} \\
\cdot\cdot\cdot & & 91 \\
 & & \underline{80} \\
 & & 11
\end{array}$$

The Reaſon of this Rule.

In the Fraction $\left(\frac{6331}{16}\right)$, 6331 ſignifying ſo many 16ths, therefore every 16 of them make an Unit, : So that the Number of times that 16 is contained in 6331 (which is found by Diviſion to be 395, and 11 over) will make a Number $395\frac{11}{16} = \frac{6331}{16}$.

PROP. VIII.

To Reduce two Fractions, having unequal Denominators into two Fractions of the ſame value, which ſhall have a common Denominator.

RULE.

Multiply each *Numerator* by the others *Denominator* for new *Numerators*, and their *Denominators* by each other for a *common* Denominator.

If the Fractions be in the *leaſt* terms, and the Denominators be not *primes*, divide the *Denominators* by their *greateſt* common meaſure, and then multiply each *Numerator* croſs-wiſe by the *Quotients* for *new* Numerators, and *either* of the Denominators by the *others* Quotient for a *common* Denominator, and then they will be in the *leaſt* terms that can be with a *common* Denominator.

Example 1.

Let the given Fractions be $\frac{3}{4}$ and $\frac{2}{5}$

$$\frac{3}{4} \times \frac{2}{5}$$

The *new* Numerators 15 8

The *common* Denominator 20

Example 2.

Let the Fractions given be $\frac{5}{12}$ and $\frac{7}{18}$.

$$\begin{array}{ccc} 6)\frac{5}{12} & \times & 6)\frac{7}{18} \\ 2 & & 3 \end{array}$$

$$\left.\begin{array}{lcc}\text{The \textit{new} Numerat.} & 15 & 14 \\ \hline \text{The \textit{common} Denom.} & \multicolumn{2}{c}{36}\end{array}\right\}\begin{array}{l}\text{in the leaſt}\\ \text{terms, with}\\ \text{a \textit{com.}Deno.}\end{array}$$

Demonſtration.

In the *firſt Example* the Operation is in effect, thus $\frac{3\times5}{4\times5}=\frac{3}{4}$ and $\frac{2\times4}{5\times4}=\frac{2}{5}$ (by *Cor.* 2.*ch.*11.) and *ſince* according to the Nature of Multiplication, $4\times5=5\times4$ (= 20 the common Denominator), *therefore* either of theſe two Multiplications *alone* will be ſufficient.

In the 2*d.* *Examp.* the Operation is in effect, *thus* $\frac{5\times3}{12\times3}=\frac{5}{12}$ and $\frac{7\times2}{18\times2}=\frac{7}{18}$ and $12\times3=18\times2=36$ the *common Denominator*, and the *reaſon* of this is the ſame with the aforegoing inſtance, only there remains to be proved that if the *Denominators* be divided by their *greateſt* common meaſure, and multiplied by their Quotients *croſs-wiſe*, the Products will be *equal*, and the Fractions in the *leaſt* terms having a *common* Denominator.

Now that a *common* Diviſor to two Numbers will divide both into the *ſame* Number of *equal* parts, is evident from the Nature of *Diviſion*, and that therefore the Products of *each* Number multiplied by *one* of the others parts are *equal*, appears as followeth,

$$\left.\begin{matrix}6)12=2\\6)18=3\end{matrix}\right\}\text{ſo that in}\left\{\begin{matrix}12\\18\end{matrix}\right\}\text{is } 6\ \textit{times}\left\{\begin{matrix}2\\3\end{matrix}\right.$$

$$\left\{\begin{matrix}2\\2\\2\\2\\2\\2\end{matrix}\right\}\times3=\left\{\begin{matrix}3\\3\\3\\3\\3\\3\end{matrix}\right\}\times2$$

$$\overline{12}\times3=\overline{18}\times2$$

By this it is evident that in *both* the Multiplications there are the *ſame* Number of *particular equal* Products (2×3=3×2) and therefore the *Total* Products will be *equal*. But if the Numbers be *not* divided by a *common* meaſure, the Number of parts in each will be *unequal*, ſo that there will be an *unequal* Number of *particular equal* Products, and conſequently the *Total* Products will be alſo *unequal*.

$$\text{As if}\left\{\begin{matrix}14\\15\end{matrix}\right\}\text{be divided by}\left\{\begin{matrix}7\\5\end{matrix}\right\}\text{its parts will be}\left\{\begin{matrix}7\\5\end{matrix}\right\}\text{times}\left\{\begin{matrix}2\\3\end{matrix}\right\}$$

$$\left\{\begin{matrix}2\\2\\2\\2\\2\\2\\2\end{matrix}\right\}\times 3 = \left\{\begin{matrix}3\\3\\3\\3\\3\\\cdot\\\cdot\end{matrix}\right\}\times 2$$

$$14\times 3 \sqsupset 15\times 2$$

For ſince 2x3=3x2, and in the *firſt* Multiplication there are 7 *times* 2x3, and in the *ſecond* but 5 *times* 3x2, therefore the *Total* Product of the *firſt* muſt be *greater* than that of the *ſecond.* So that the Denominators muſt have a *common* Diviſor to make their Products *equal*, and to bring Fractions into the *leaſt* termes it muſt be the *greateſt* common Diviſor. *W. W. D.*

PROP. IX.

To find a Number that being divided by 2 *or more Numbers given ſhall have no remainder.*

RULE.

If the Numbers given be *only two* multiply the one by the other, or if they be *more* than *two*, multiply them *continually* in any order, and the *Product* will be the Number *ſought.*

Example 1.

Let the Numbers given be 17 and 19.

Multiply 19
by 17
133
19
And the *Product* is 323 the Number *ſought*.

Example 2.

Let the Numbers given be 5. 3. 7. 9.

Multiply 5 or 7
by 3 by 5
Multiply the *Product* 15 or 35
by 7 by 9
And this *Product* 105 or 315
by 9 by 3
The Number *ſought* 945 945

Demonſtration.

For by the nature of Multiplication the ſame Numbers, in what order *ſoever* they be multiplyed, will have the *ſame* Product. As in the laſt inſtance, ſuppoſing $5\times3\times7\times9 = P$.

Then

$$\text{Then will}\left\{\begin{matrix}3\times7\times9\\5\times7\times9\\5\times3\times9\\5\times3\times7\end{matrix}\right\}\times\left\{\begin{matrix}5\\3\\7\\9\end{matrix}\right\}=P=945$$

whereby it appears that 945 is a *common* Dividend to the Numbers 5. 3. 7. 9. without *any* remainder. For *Diviſion undoes what is done by Multiplication.*

P R O P. X.

To find the leaſt Number, that being divided by as many Numbers as you pleaſe, ſhall have no Remainder.

R U L E.

Take *any* two Numbers given, and if they be *primes*, multiply the one by the other, but if they be *not*, divide them by their *greateſt common* meaſure, and multiply either of them by the others Quotient, then take the Product as *one*, and any one of the remaining Numbers as *another*, and work with theſe as with the two Numbers *firſt* taken, and ſo proceed with all the reſt, and the *laſt Product* will be the Number *required.*

Example 1.

Let the Numbers be 3. 15. 11.

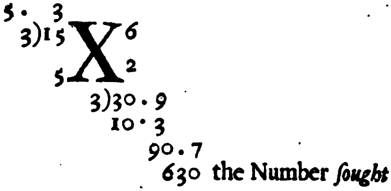
$$\begin{array}{rcc}3)\ 3 & \times & 15\\ 1 & & 5\end{array}$$

$$\begin{array}{r}15.11\\15\\\hline 165\end{array}$$

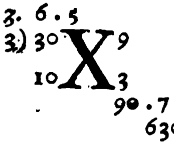
The Number *ſought* 165

Example 2.

Let the Numbers given be 5. 3. 6. 9. 7.



Note, *That if one of the two Numbers taken be an aliquot, that is, an even part of the other, it muſt be omitted, and another put in its ſtead, as in the following Operations.*



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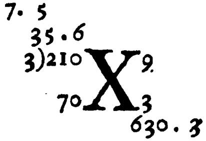


Demonſtration.

The **Reaſon** of this Rule is the ſame with the laſt, only there remains to be ſhewn how, the *laſt* Product is the *leaſt* Number that can be divided by the Numbers *given* without *any* remainder, and that may be done by explaining the two laſt Operations, in the firſt whereof 30 and 9 being divided by their greateſt common meaſure, the Product (90) of the croſs Multiplication is (as appears by *Demonſtration* in 8 *Prop.* hereof) the *leaſt* Number that will exactly contain the 2 Numbers 30 and 9, each an *exact* Number of times, *that is*, which can be divided by each of them without any remainder, and therefore 90 is to be taken *inſtead* of the Product of 30x9, and by the *ſame reaſon*, in the ſecond operation 630 *inſtead* of 210x9, and that 3 in both the operations is to be omitted, appears by the *croſs* Multiplications, they making no alterations in the Numbers 6 and 630, their Multiplier being 1, which it is evident will *alwayes* be ſo, when one of the Numbers taken is an *even* part of the other.

$$3)\begin{array}{ccc}3 & & 6\\ & \times & \\ 1 & & 2\\ & & 6\end{array} \qquad 3)\begin{array}{ccc}630 & & 3\\ & \times & \\ 210 & & 1\\ & & 630\end{array}$$

PROP. XI.

To Reduce any Number of Fractions, having unequal Denominators to Fractions of the ſame value, which ſhall have a common Denominator.

RULE 1.

Firſt find, *as* by *Prop.* 9, or (if you would have the Fractions in the *leaſt* terms by *Prop.* 10) a Number which may be meaſured by *all* the Denominators, and that will be the *common* Denominator, then Divide this Denominator by each of the Denominators given, and multiply the Quotients by the Numerators *given* for *new* Numerators reſpectively.

Examp.

Let the Fractions be $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{6}{7}$.

First, According to the Rule in *Prop.* 9.

2 , 3
6 , 4
24 , 5
120 , 6
720 , 7
5040 the Common *Denominator.*

Then,

Divisors, or Denom. given.	*Dividend*	*Quot.*	(*Numerators given*) =	The new Numerators to the Denominator 5040.
2	5040	2520 × 1	=	2520
3		1680 × 2	=	3360
4		1260 × 3	=	3780
5		1008 × 4	=	4032
6		840 × 5	=	4200
7		720 × 6	=	4320

Secondly more *briefly*, according to the Rule in *Prop.* 10.

2 , 3
6 , 4
3 , 2
12 , 5
60 , 6 , 7
420 The *least common* Denominator to the Fractions given, if in the *least* terms.

Then,

Diviſors, or *Denominators* given.	*Dividend*	*Quot.*	(*Numerators* given) =	The new Numerator to the *common* Denominator.
2	420	210×1		210
3		140×2		280
4		105×3		315
5		84×4		336
6		70×5		350
7		60×6		360

Demonſtration.

It is manifeſt, (that according to this Rule) the *common Denominator* may be *exactly* divided by each *Denominator* of the *Fractions* given, and that therefore *ſuch* a part or parts thereof may be taken *exactly* as each Fraction given, is (ſuppoſed to be) of an *Unit*. Thus (according to the laſt Operation) the common Denominator is 420 (*that is*, the Unit is ſuppoſed to be divided into 420 equal parts) and therefore the one half of theſe parts (that is, $\frac{210}{420}$) is equal to $\frac{1}{2}$ of an Unit, and this is found by dividing 420 by 2, and multiplying the Quotient by 1, and thus, if 420 be divided by 3, the Quotient will be 140 (1 *third* of 420) which multiplied by 2 will make 280, (*two thirds* of an Unit divided into 420 equal parts) and therefore of the ſame value with *two thirds* of an Unit, and ſo for the reſt.

RULE 2.

Multiply each Numerator continually by the Denominators of the remaining Fractions for new Numerators, and

and the Denominators continually for a common Denominator.

Example.

Let the *ſame* Fractions be given as *before*, $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{6}{7}$.

1	2	3	4	5	6
3	2	2	2	2	2
3	4	6	8	10	12
4	4	3	3	3	3
12	16	18	24	30	36
5	5	5	4	4	4
60	80	90	96	120	144
6	6	6	6	5	5
360	480	540	576	600	720
7	7	7	7	7	6
2520	3360	3780	4032	4200	4320

Theſe *laſt* Products are the *New* Numerators.

2
3
———
6
4
———
24
5
———
120
6
———
720
7
———
5040 the *common* Denominator.

Demonſtration.

By this Method of Operation *each* Numerator and *each* Denominator of each Fraction is in effect continually multiplied by the remaining Denominators; and therefore (by *Cor.2. chap.*11.) reduced into Fractions of the *ſame* value. And ſince in what order ſoever the Denominators be multiplied, the Product will be the ſame, therefore there will be no neceſſity of multiplying them any more than once, *that is*, according to any other order than that they are given in, and this Product will be the *common Denominator*.

CHAP. XIII.

Addition *of* Vulgar Fractions.

RVLE 1.

IF the Fractions have equal Denominators, add their Numerators together for a new Numerator, and to the Sum ſubſcribe the ſame Denominator, and this new Fraction will be the Total, but if the Fractions given have unequal Denominators, let them be firſt reduced (as by Prop. 8. 11. chap. 12.) *to ſuch as ſhall have equal Denominators.*

Example 1.

Let the Fractions given be $\frac{13}{17}$ and $\frac{9}{17}$.

To 13
Add 9

The Total $\frac{22}{17} = 1\frac{5}{17} = \frac{13}{17} + \frac{9}{17}$

Examp.

Example 2.

Let the Fractions given be $\frac{19}{31}\ \frac{17}{23}$

$$\frac{19}{31} \times \frac{17}{23}$$

$$\left.\begin{array}{l} 19\times23=\left\{\begin{array}{r}57\\38\end{array}\right.\\[4pt] 17\times31=\left\{\begin{array}{r}17\\51\end{array}\right.\\ \hline 23\times31=\left\{\begin{array}{r}23\\69\end{array}\right.\end{array}\right\} = \frac{964}{713} = 1\,\frac{251}{713}\ \text{the } \textit{Total}$$

Example 3.

Let the Fractions given be $\frac{5}{12}\ \frac{7}{18}$

$$6)\ \frac{5}{\underset{2}{12}} \times \frac{7}{\underset{3}{18}}$$

$$\frac{15 + 14}{36} = \frac{29}{36}\ \text{The } \textit{Total}$$

Examp. 4.

Let the Fractions given be $\frac{2}{3}\ \frac{1}{4}\ \frac{2}{5}$.

$60 = 3\times4\times5$ *that* is, all the Denom. multiplyed contin. for a *common* Denom.

$$\left.\begin{array}{l|l} \frac{2}{3} & 40 = \frac{60}{3}\times2 \\ \frac{1}{4} & 15 = \frac{60}{4}\times1 \\ \frac{2}{5} & 24 = \frac{60}{5}\times2 \end{array}\right\}$$

that is, the *common* Denominator (60) divided by the ſeveral *Denom.* and the *Quot.* multiplied by their reſpective *Numerators* for *new Numerators.*

The Total $\frac{79}{60} = 1\frac{19}{60}$.

Examp. 5.

Let the Fractions given be $\frac{1}{2}\ \frac{2}{3}\ \frac{3}{4}\ \frac{4}{5}\ \frac{5}{6}\ \frac{6}{7}$.

2 , 3
6 , 4
3 , 2
12 , 5
60 , 7
420 The leaſt *common* Denominator to the Fractions given, if in the *leaſt* termes.

$$\left.\begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}\right\} 420 \left\{\begin{array}{l} 210 \times 1 = 210 \\ 140 \times 2 = 280 \\ 105 \times 3 = 315 \\ 84 \times 4 = 336 \\ 70 \times 5 = 350 \\ 60 \times 6 = 360 \end{array}\right\}$$

The *Numerators* to the common *Denominator* 420.

The *Total* $\frac{1851}{420} = 4\frac{171}{420}$.

RULE

RULE 2.

Add two of the Fractions together as before directed, and to the Sum add another Fraction, &c. and the last Fraction will be the Total.

Examp.

Let the Fractions given be as before,

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{6}{7}$.

$\frac{1}{2}$ $\frac{2}{3}$

3

4 7 3

$\cancel{6}$ $\cancel{4}$

3 2

14

9 23 4

12 5

115

48 163 5

60 6

10 1

163

50 213 6

60 7

1491

360 $\frac{1851}{420} = 4\frac{171}{420}$ The Total as before.

CHAP. XIV.

Subſtraction of Vulgar Fractions.

RULE.

IF the Fractions given have equal Denominators, ſubſtract the leſſer Numerator from the greater for a new Numerator to the ſame Denominator, and this Fraction will be the Remainder. But if they have unequal Denominators, let them be firſt reduced as by Prop. 8. chap. 12.

Examp. 1.

Let $\frac{19}{31}$ be given to be ſubſtracted from $\frac{17}{23}$.

$$\frac{17}{23} \qquad \frac{19}{31}$$

$17\times31 = \left\{ {17 \atop 51} \right\} = 527$ for a new *Numerator.*

$19\times23 = \left\{ {57 \atop 38} \right\} = \underline{437}$ for a new *Numer.* which ſubſtract

$\frac{90}{713}$ = The *Remainder.*

$23\times31 = \left\{ {23 \atop 69} \right\} = 713$ for a new *Denominator.*

Examp. 2.

Let $\frac{3}{13}$ be given to be ſubſtracted from $\frac{7}{13}$.

From 7
Subſtract 3

$\frac{4}{13}$ the Remainder.

Examp. 3.

Let $\frac{2}{5}$ be given to be ſubſtracted from $\frac{3}{4}$.

$$\frac{3}{4} \times \frac{2}{5}$$

New Numerators
common Denominat. $\frac{15 - 8}{20} = \frac{7}{20}$ the *Remain.*

Where the Fraction to be ſubſtracted is greater than that from which you are to ſubſtract it, borrow an Integer, and proceed as in whole Numbers.

Examp.

from $13\,\frac{3}{5}$
Subſtract $5\,\frac{4}{5}$

The *Remainder* $7\,\frac{4}{5}$.

Here $\frac{4}{5}$ cannot be ſubſtracted from $\frac{3}{5}$ Therefore borrowing an Integer (that is, 5 *fifths*) ſay 4 from 5, and there remains 1, which with the 3, makes 4, *that is*, $\frac{4}{5}$, 1 borrowed and 5 make 6, which ſubſtracted from 13, there remains 7.

The **Reaſon** of the Rules in *Addition* and *Subſtraction* of Vulgar Fractions is very plain. For when *Fractions* have the ſame *Denominator*, they then have the *ſame* Name. And therefore as 5^{yds} and 3^{yds} make 8 *yards*, ſo 5 *Thirteenths*, and 3 *Thirteenths* make 8 *Thirteenths*. And 3 *Thirteenths* ſubſtracted from 5 *Thirteenths* leave 2 *Thirteenths*.

Note, That *like* terms of Fractions ſignifie either their *Numerators* or *Denominators* and *Alternate* terms of two Fractions ſignifie either the *Numerator* of the one and the *Denominator* of the other, or the *Denominator* of the one and the *Numerator* of the other.

CHAP. XV.

Multiplication *of* Vulgar Fractions.

RULE.

MUltiply the Numerators by each other for a new Numerator, and likewiſe the Denominators for a new Denominator: But if the Alternate termes be not prime, divide them by their greateſt common

common meaſure, and uſe the Quotients in their ſtead, and the Fraction produced will be in the leaſt termes.

Examp. 1.

Let $\frac{2}{5}$ be multiplied by $\frac{3}{7}$.

$\frac{2\times3}{5\times7} = \frac{6}{35}$ The *Product*.

Examp. 2.

Let $\frac{7}{45}$ be multiplyed by $\frac{5}{21}$.

$$\begin{array}{l} \quad\ 1 \times 1 \\ 7)\frac{\cancel{7}}{\cancel{45}} \times \frac{\cancel{5}}{\cancel{21}} = \frac{1}{27} \\ 5)\ \ 9 \times 3 \end{array}$$

The *Product* in the leaſt *terms*.

Demonſtration:

To *multiply* a Fraction by a Fraction is (by the Nature of *Multiplication*) to *take* the *Multiplicand* ſo many *ſuch parts* of a time as the *Multiplier* ſignifies. Now the *Denominator* of the Multiplier ſhews *what* part of the *Multiplicand* is to be taken, *that is*, that it muſt be *divided* by the ſaid *Denominator*, and this (by *Cor.*1.*chap.*11.) is done by *multiplying* the *Denominator* of the Multiplicand by that of the *Multiplier*: And ſince the *Numerator* of the *Multiplier* ſhews how many *ſuch parts* of the *Multiplicand* are to be taken; therefore (by the ſame *Cor.*) the *Numerator* of *this* muſt be multiplied by the *Numerator* of *that*.

In the firſt Example (from the Nature of Multiplication) it is evident that to multiply $\frac{2}{5}$ by $\frac{3}{7}$, is to take 3 ſevenths of $\frac{2}{5}$ and (according to the notion of Diviſion) it appears that to take 1 ſeventh of $\frac{2}{5}$ is to divide $\frac{2}{5}$ by 7, and the Quotient (by Cor. 1. chap. 11.) will be $\frac{2}{5\times7}$ and therefore 3 ſevenths of $\frac{2}{5}$ will be thrice $\frac{2}{5\times7}$, that is, (per ſame Cor.) $\frac{2\times3}{5\times7} = \frac{2}{5}\times\frac{3}{7}$. W. W. D.

In the ſecond Examp, if $\frac{7}{45}$ be multiplyed by $\frac{5}{21}$ according to the Operation of the 1ſt. Examp. the Product will be $\frac{7\times5}{45\times21}$ in which 7 and 21 (the alternate terms of $\frac{7}{45}$ and $\frac{5}{21}$) being divided by 7 (their greateſt common meaſure), the Products of 7×5 and 45×21 (that is both the termes of this Fraction $\frac{7\times5}{45\times21}$) will in effect be divided by 7, and therefore (by Cor. 2. chap. 11.) produce another Fraction $\left(\frac{1\times5}{45\times3}\right)$ of the ſame value, in which 5 and

45 (the *remaining alternate* termes of $\frac{7}{45}$ and $\frac{5}{21}$) being *divided* by 5 (their *greateſt common* meaſure) there will alſo by the ſame reaſon ariſe *another* Fraction $\left(\frac{1\times 1}{9\times 3}\right)$ of the *ſame* value. All which may *briefly* and *plainly* appear as followeth.

$$\frac{7)\overset{1}{\cancel{7}}\times 5}{45\times\underset{3}{\cancel{21}}} = \frac{1\times\overset{1}{\cancel{5}}}{5)\underset{9}{\cancel{45}}\times 3} = \frac{1\times 1}{9\times 3} = \frac{1}{27} = \frac{7}{45}\times\frac{5}{21}$$

If a Fraction $\left(\frac{5}{28}\right)$ be given to be *multiplied* by a *whole* Number (4) either *multiply* the *Numerator*, or (if it can be done without any remainder) *divide* the *Denominator* by the *whole Number*,

Then,

$$\left.\begin{array}{r}\frac{5\times 4}{28}=\frac{20.}{28}\\ \text{or}\\ 4)\frac{5}{28}=\frac{5}{7}\end{array}\right\}=\frac{5}{28}\text{ multiplied by 4, (per Cor. 1. ch. 11.)}$$

If the Numbers to be multiplied be *mixt*, let them be firſt reduced, (as by *Prop.* 4. *chap.* 12.) and then work as before directed.

Examp.

Let $7\frac{3}{4}$ be *multiplyed* by $5\frac{1}{7}$.

$$\left.\begin{array}{r} 7\frac{3}{4} \\ 5\frac{1}{7} \end{array}\right\} \textit{reduced}\text{ make} \left\{\begin{array}{l} \frac{31}{4} \\ \frac{36}{7} \end{array}\right.$$

$$\text{Then } \frac{31}{4} \substack{\times \\ +} \frac{36}{7} \substack{= \\ =} \frac{1116}{28} \left(=39\frac{6}{7}\right) = 7 \times 5\frac{6}{7}$$

But if *one* of the Numbers be *mixt*, and the *other* a *whole* Number, as $7\frac{3}{4}$ and 9, the Operation may be performed without *any* reduction, as followeth.

$$\begin{array}{rr} \text{Multiply} & 7\frac{3}{4} \\ \text{by} & 9 \\ \hline \text{The } \textit{Product} & 69\frac{3}{4} \end{array}$$

Here 9 *times* 3 is 27, 4 of which make an Unit (becauſe the *Denominator* is 4) and *therefore* there being 6 *times* 4 in 27 and 3 over, 6 muſt be carried, and 3, *that is*, $\frac{3}{4}$ ſet underneath, and 9 *times* 7 with the 6 make 69.

☞ *Note*, That two *proper* Fractions being given, to *multiply* this by *that*, or that by *this*, is to *take* neither ſo much as *once*, a proper Fraction being *leſs* than an Unit, and conſequently the Product, which is either way the ſame, will be *leſs* than *either* the *Multiplier* or *Multiplicand*. As to *multiply*

multiply $\frac{3}{4}$ by $\frac{2}{5}$, is to *take* $\frac{3}{4}$ not ſo *much* as once, *that is*, but $\frac{2}{5}$ of once: Or to *multiply* $\frac{2}{5}$ by $\frac{3}{4}$ is to take not *once* $\frac{2}{5}$ but ſo many *parts* thereof as are leſs than the whole. So that the *Product* $\left(\frac{3}{4}\times\frac{2}{5}\text{ or }\frac{2}{5}\times\frac{3}{4}=\frac{6}{20}\right)$ will be *leſs* than either $\frac{3}{4}$ or $\frac{2}{5}$.

CHAP. XVI.

Diviſion of Vulgar Fractions.

RULE.

Multiply the Numerator of the Dividend by the Denominator of the Diviſor for a new Numerator, and likewiſe the 2 remaining termes by each other for a new Denominator. Or if like termes be not prime, divide them by their greateſt common meaſure, and uſe the Quotients in their ſtead, and the Fraction produced will be the Quotient in the leaſt termes.

Examp. 1.

Let $\frac{3}{7}$ be given to be divided by $\frac{2}{5}$.

$\frac{2}{5}\Big)\frac{3}{7}\Big(\frac{15}{14}\Big(=1\frac{1}{14}\Big)=\frac{3}{7}$ *divided* by $\frac{2}{5}$.

Examp. 2.

Divide $\frac{9}{32}$
by $\frac{3}{8}$

$$\begin{array}{l} 3) \\ 8) \end{array}\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}}\Bigg)\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{32}}}\Bigg(\frac{3}{4}$$

the *Quotient* of $\frac{9}{32}$ *divided* by $\frac{3}{8}$ in the *leaſt termes.*

Demonſtration.

To *divide* a Fraction by a Fraction is (by the Nature of *Diviſion*) to find how oft the *Diviſor* (*that is*, by the 2*d.* Notion of a Fraction in Chap. 11, how oft *ſuch* a *part* of its *Numerator* as is expreſt by the *Denominator*) is *contain'd* in the *Dividend*, and ſince that will be *ſo* many times *oftner* than the *Numerator* is contain'd therein; *as* there are *Units* in the *Denominator*, becauſe the Diviſor being a *Fraction* is ſo many times *leſs*. Therefore it is evident (by *Cor.* 1. *chap.* 10.) that if the *Denominator* of the *Dividend* be multiplied by the *Numerator* of the *Diviſor*, and likewiſe the *Numerator* of *this*, by the *Denominator* of *that*, the Fraction thence ariſing will be the *true Quotient.*

Thus,

In the firſt Examp. to *divide* $\frac{3}{7}$ by $\frac{2}{5}$ is to find out how oft $\frac{2}{5}$ is *contain'd* in $\frac{3}{7}$. Now to find out how oft 2 (the *Numerator* of the *Diviſor*) is *contain'd* in $\frac{3}{7}$ or (which is the ſame thing) to *divide* $\frac{3}{7}$ by 2 is done (by *Cor.* 1. *chap.* 11.) by *multiplying* the *Denominator* 7 by 2 (that is $\frac{3}{7\times2}$) but ſince the Diviſor $\left(\frac{2}{5}\right)$ is but a *fifth* part of 2, it will be *contain'd* in $\frac{3}{7}$ 5 *times* oftner than 2 is, and therefore the *Quotient* $\left(\frac{3}{7\times2}\right)$ muſt be 5 *times* greater, *that is*, the *Numerator* 3 muſt be multiplied by 5 (the *Denominator* of the Diviſor); As $\frac{3}{3\times5}\times5=\frac{3\times5}{7\times2}$: So that $\frac{3\times5}{7\times2}\left(=\frac{15}{14}=1\frac{1}{14}\right)$ will be the *true Quotient.* *W. W. D.*

In Examp. 2. if $\frac{9}{32}$ be *divided* by $\frac{3}{8}$ according to the Operation in the firſt *Examp.* the *Quotient* will be $\frac{8\times9}{3\times32}$, ſo that (as by Demonſtration of *Examp.* 2. *chap.* 15.)

$$3)\,\frac{8\times\overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}}\times32}=8)\,\frac{\overset{1}{\cancel{8}}\times3}{1\times\underset{4}{\cancel{32}}}=\frac{1\times3}{1\times4}=\frac{3}{4}=\frac{9}{32}\text{ divided by }\frac{3}{8}$$

If a *Fraction* $\left(\frac{6}{7}\right)$ be given to be *divided* by a *whole* Number (3) it may be done either by *multiplying* the *Denominator*, or (if it can be without any Remainder) by *dividing* the *Numerator* by the *whole Number* (as appears by *Cor.*1. *chap.*11.) Thus,

$$\left.\begin{array}{l} \frac{6}{7\times3} = \frac{6}{21} \\[1em] 3)\ \frac{6}{7} = \frac{2}{7} \end{array}\right\} = \frac{6}{7} \text{ divided by } 3.$$

If the *Diviſor* or *both* the Numbers given be *mixt*, let them be firſt *reduced* (as by *Prop.* 4. 5. *chap.* 12.) and then work as before *directed.*

Examp. 1.

$$\left.\begin{array}{r} \text{Divide } 3\,\frac{1}{3} \\[1em] \text{by } 2\,\frac{1}{2} \end{array}\right\} \text{which reduced make } \left\{\begin{array}{l} \frac{10}{3} \\[1em] \frac{5}{2} \end{array}\right.$$

$$\text{Then, } 5)\ \frac{\overset{1}{\cancel{5}}}{2}\Big)\frac{\overset{2}{\cancel{10}}}{3}\Big(\frac{4}{3}\Big(= 1\,\frac{1}{3}\Big) \text{ the Quotient.}$$

Examp. 2.

$$\left.\begin{array}{r} \text{Divide } 35 \\[1em] \text{by } 2\,\frac{2}{3} \end{array}\right\} \text{which reduced make} \left\{\begin{array}{l} \frac{105}{3} \\[1em] \frac{8}{3} \end{array}\right.$$

$$3)\,\frac{8}{\underset{1}{3}}\Big)\frac{105}{\underset{1}{3}}\Big(\frac{105}{8}\Big(= 13\,\frac{1}{8}\Big)\text{the } \textit{Quotient}$$

From this laſt inſtance it appears that if the *Fractions* have *equal* Denominators, the *Diviſion* is performed by placing the *Numerator* of the *Dividend* over the *Numerator* of the *Diviſor*.

If a *mixt* Number be given to be *divided* by a *whole* Number, as $379\,\frac{2}{7}$ by 5, the Operation may ſometimes more conveniently be performed as followeth.

$$5)\ 379\,\frac{2}{7}\ (75\,\frac{6}{7}$$

Here 5 is contained in 379 (the *Dividend*) 75 *times* and 4 Units over, which multiplyed by 7 (every Unit, in this caſe, being 7 *ſevenths*) makes 28, this and 2 make 30, that is, 30 *ſevenths*, in which 5 is contained 6 *times*, or the 5th. *part* of 30 *ſevenths* is $\frac{6}{7}$.

In dividing *proper* Fractions by each other, the *Dividend* being in effect the *Product* of the *Diviſor* and *Quotient* multiplied together, will (as hath been proved in Multiplication) be *leſs* than either of them, when the *Quotient* is a *proper* Fraction. Or in dividing *any* Fraction or *whole* Number by a *proper* Fraction, the *Quotient* will alwayes be *greater* than the *Dividend*. For it is evident, that when a Number is divided by an *Unit* or 1, the *Quotient* will then be *equal* to the *Dividend*: And therefore, when it is divided by a *proper Fraction* (which is *leſs* than an *Unit*) the *Quotient* will then be greater than the *Dividend.* For *a leſſer Diviſor makes a greater Quotient.*

As, $\frac{2}{3}\big)\frac{5}{7}\Big(=\frac{5\times3}{7\times2}=\frac{15}{14}\sqsupset\frac{5}{7}$

For $1)\frac{5}{7}\Big(=\frac{5}{7}$ but $\frac{2}{3}\sqsupset 1$ *Ergo* the Quotient of $\frac{5}{7}$ Divided by $\frac{2}{3}$ will be $\sqsupset 1)\ \frac{5}{7}$ (*that is,* $\frac{5}{7}$). *W. W. D.*

PROP. I.

To Reduce a Vulgar Fraction into a Decimal.

RULE.

Add *one* or *more* Cyphers to the Numerator, and then divide it by the *Denominator* (as directed in *chap.* 10.) and the *Qutient* (if nothing remain after the *Diviſion*) will be a *Decimal* of *equal* value with the *Vulgar* Fraction: Or (if there be a remainder) by adding more Cyphers you may proceed ſo as to make the Quotient (*near*) equal.

Examp.

$\frac{3}{4} = \frac{3.00}{4} = .75$ $\frac{1}{8} = \frac{1.000}{8} = .125$

$\frac{2}{17} = \frac{20000}{17}$, &c. $17)2.0000(.11764$ *near equal to* $\frac{2}{17}$

17 . . .

30

17

130 not wanting $\frac{1}{100000}$ *part of an Unit.*

119

110

102

80

68

12

PROP. II.

To Reduce a Decimal into a Vulgar Fraction.

RULE.

Divide the *Decimal* and an *Unit* with as many Cyphers put to it as there are Decimal places, by 5 *continually*, when the Decimal ends with 5, and the Quotients will be the *termes* of a *vulgar* Fraction *equal* to the Decimal *given*: But if it do *not* end with 5, Divide the terms by their *greater common* meaſure.

Examp.

$$\frac{5).975}{5)1000}\Big|\frac{195}{200}\Big|\frac{39}{40} = .975 \qquad \frac{5).25}{5)100}\Big|\frac{5}{20}\Big|\frac{1}{4} = .25$$

$$\frac{5).5}{5)10} = \frac{1}{2} \qquad \frac{5).75}{5)100}\Big|\frac{15}{20}\Big|\frac{3}{4} = .75 \qquad \frac{4).12}{4)100}\Big(\frac{3}{25} = 12$$

$$\frac{5).125}{5)1000}\Big|\frac{25}{200}\Big|\frac{5}{40}\Big|\frac{1}{8} = .125$$

The Reaſon of this Rule will eaſily appear by conſidering a Decimal as a Fraction, whoſe Numerator is the Decimal, and its Denominator an Unit with as many Cyphers as there are places in the Decimal.

As $.37$ is $\frac{37}{100}$ $.053$ is $\frac{.053}{1000}$ or $\frac{53}{1000}$.

Arithmetick in Species:

OR,

ALGEBRA.

BOOK II.

Arithmetick in Species:

OR,

ALGEBRA.

CHAP. I.

Algebraic Notation.

THE *method of reſolving* Queſtions in Arithmetick is either *Analitical*, that is, the *taking* of that which is *enquired after*, *as if it were* known, and thence going back by conſequences until it be *found* to be equal to ſomething *really* known, or elſe *contrariwiſe Sinthetical*, *that is*, the *taking* of that which is *granted* or *known*, and thence proceeding by conſequences to the *diſcovery* of what is *enquired after*, and *unknown*.

The *Analitical* art of reſolving difficult Queſtions in Arithmetick is commonly called **Algebra.**

In *Algebraic* Arithmetick Quantities or Numbers are repreſented by *Letters*, and ſometimes by *figures* alſo, (as there may be occaſion.) The *Advantage* of *thus expreſſing* Quantities is that the *Numbers with their ſeveral Operations*, which in *vulgar* Arithmetick would be *loſt* or *ſwallowed up*, in *Algebraic* Arithmetick are ſo order'd and managed as to be *preſerved diſtinct* and *in view*, and at laſt to produce a *Canon* or *Rule* for reſolving not only the *particular* Queſtion *propoſed* but in general any *other* of the like Nature.

Beſides theſe *Letters* there are therewith uſed certain *ſigns* which are as followeth.

An Explanation of the Signs uſed in Algebra.

+	ſignifies	More or added to
—		Leſs or ſubſtracted from
×		Multiplied by, or multiplying
÷		Divided by, or dividing
÷)		Continually divided by
=		Equal to
$\surd$ or $\surd^{2}$		The Square Root, or the Root of the 2d. power.
∺		Continual Geomet. Proportion
::		Disjunct Geomet. Proportion
∻		Continual Arithmet. Proportion
∵		Disjunct Arithmet. Proportion
⊐ or >		Greater than
⊏ or <		Leſs than
∽		The difference of two Quantities, when it is not known which of them is the greater.
∴		Therefore

$b+c$	ſignifies	b more c or c added to b.
$b-c$		$b-c$ or c ſubſtracted from b.
$b\times c$ or bc		b multiplied by c, or b multiplying c.
$b \div c$ or $\frac{b}{c}$		b divided by c, or c dividing b
$b=c$		b equal to c.
$\sqrt{b}$ or $\sqrt{2}\,b$		The Squ. Root of b, or the quantity of which b is the 2*d*. Pow.
$a.\,b.\,c.\,d\ \div\!\!\div$		As a is to b, ſo b to c, ſo c to d.
$a:b::c:d$		As a is to b, ſo is c to d.
$a.\,b.\,c.\,d \div$		$a.\,b.\,c.\,d.$ have equal differences
$a:b \div c:d$		The difference of a and b is equal to that of c and d.
$b \sqsupset c$ or $b > c$		b greater than c
$c \sqsubset b$ or $c < b$		c leſs than b
$b \backsim c$		$b-c$ or $c-b$ according as b or c is greater.

The *former* of the ſigns (+ —) is a note of *Poſition* or *Affirmation*, the other of *Defect* or *Negation*, the *one* ſuppoſes the Quantity following it to *be*, the *other* that it is *wanting* or *not* to *be* (and when a quantity hath *no* ſign before it, it is generally ſuppoſed to be *Affirmative*, and the ſign + to be underſtood).

And thus theſe two ſigns muſt always have a *contrary* ſignification. If + ſignifie *Upward*, *Forward*, *Gain*, *Encreaſe*, *Above*, *Before*, &c. then — muſt ſignifie *Downward*, *Backward*, *Loſs*, *Decreaſe*, *Below*, *Behind*, &c. And if + ſignifie *theſe*, then — muſt ſignifie the *contrary*.

A Number *prefixt* to any Quantity ſhews how oft it is taken, as 6*d*, 9*b*, *&c.* ſignifies 6 *times d*, 9 *times b*, &c.

A Number ſet at the *head* of a Letter ſhews the *power* or degree of that Quantity, and at the *head* of a Radical Sign denotes the *Root* of ſuch a power, and this Number is called the *Index*.

The Method of expreſſing ſimple powers, *and the* Roots *of* Powers.

a	or	a	ſignifies the	1ſt	power of a.
aa		a^2		2d	
aaa		a^3		3d	
$aaaa$		a^4		4th	
$aaaaa$		a^5		5th	
$aaaaaa$		a^6		6th	
$aaaaaaa$		a^7		7th	
		&c.			

$\sqrt{a}$ or $\sqrt[2]{d}$	ſignifies the Root or Quantity whoſe	2d	power is d or ſignifies the extraction of the Root of theſe Powers.
$\sqrt[3]{d}$		3d	
$\sqrt[4]{d}$		4th	
$\sqrt[5]{d}$		5th	
$\sqrt[6]{d}$		6th	
&c.		&c.	

Quantities expreſt by *Letters*, are either *ſimple* or *compound*.

Simple Quantities are ſuch as are expreſt by a *ſingle* Letter, or by *more* join'd together like Letters in a word, as a, ab, &c.

Compound Quantities conſiſt of two or more *ſimple* Quantities join'd together with ſigns, as $a-b$, $c+d$, $cb+a+e$, &c.

When a *compound* of divers quantities is *jointly* conſider'd as *one*, it is expreſt either by putting them between two Colons, or by drawing a line over them. As $\surd : aa+ba :$ or $\surd\overline{aa+ba}$ ſignifies the Square Root of $aa+ba$ conſider'd as one Quantity. And $: c+d+e : \times f$ or $\overline{c+d+e}\times f$ ſignifies that the whole compound Quantity $c+d+e$ is multiplied by f.

Like Quantities are thoſe which are expreſt by the ſame Letters under the ſame power, as b and b, a^2 and a^2, &c.

Unlike Quantities are ſuch as are expreſt by *different* letters, or by the *ſame* letters under *different* powers. As a and b, b^3 and b^5, *&c.*

Algebraick *Integers* are ſuch Quantities as are not expreſt Fraction-wiſe. As a, $a+b$, &c.

Algebraic *Fractions* are ſuch as are expreſt after the manner of Vulgar Fractions. As $\frac{b}{c}$ $\frac{c+b}{g+a}$ &c.

Axioms.

1. *If Equal Quantities be added to, or ſubſtracted from equal Quantities, their Totals or Remainders will be equal.*

2. *If equal Quantities be multiplied or divided by equal Quantities, their Products or Quotients will be equal.*

3. *The Squares, Cubes, &c. of equal Quantities are equal.*

4. *The Square, Cube, &c. Roots of equal Quantities are equal.*

5. *Thoſe Proportions which are equal to the ſame Proportion, are equal to one another.*

CHAP. II.

Addition *of* Algebraic Integers.

RULE.

Conjoyn the Quantities propoſed, preſerving the Signs.

Examp.

To	$3a$	a	$3a$	a	$a+b$
Add	a	$-a$	$-5a$	e	$a-b$
Sum	$3a+a$	$a-a$	$3a-5a$	$a+e$	$a+a+b-b$
that is,	$4a$	0	$-2a$		$2a$

To	$a+b$	abc
Add	$a-c$	$2abc$
Sum	$a+a+b-c$	$abc+2abc$
that is	$2a+b-c$	$3abc$

$$\text{To be add.}\left\{\begin{array}{l}\left.\begin{array}{r}7d\\4d\end{array}\right\}=14d\\ \left.\begin{array}{r}-3d\\-2d\end{array}\right\}=-5d\end{array}\right.\quad\Bigg|\quad\left.\begin{array}{r}2gg\\-5gg\\4gg\\-3gg\end{array}\right\}\text{that is}\left\{\begin{array}{l}\left.\begin{array}{r}2gg\\4gg\end{array}\right\}=+6gg\\ \left.\begin{array}{r}-5gg\\-3gg\end{array}\right\}=-8gg\end{array}\right.$$

The *Sum* is $+9d$ | The *Sum* is ——— $-2gg$

When *like* Quantities have *like* ſigns, the **Reaſon** of adding them together (as is done in the foregoing Examples) is evident from the Principles of common Arithmetick. For as 2 and 3 make 5 in *all* things of the *ſame* kind, ſo will 2 a's and 3a's make 5a's, and by the ſame reaſon if to the want of 3a's (or 3 *defects* of a) be added the *want* of 2a's (or 2 *defects* of a) the *total* will be the want of $5a$'s (or 5 *defects* of a) *that is*, if to $-3a$ be added $-2a$ the ſum will be $-5a$. But the caſe is ſomewhat *different* where the ſigns are *contrary*. As if to $5a$'s (or $+5a$'s) you add $-3a$'s it makes $5a-3a$ or $5a$'s *wanting* $3a$'s which makes $2a$'s. For it is in effect the *putting* or *ſubjoyning* to $5a$'s nothing *leſs* by $2a$'s or nothing *except* $2a$'s, which is the *taking away* $2a$'s: For *ſo* much as is *taken* from any Quantity *ſo* much it *wants* of having nothing *added* to it. And if to $3a$'s be added *leſs* $5a$'s, *that is*, $3a-5a$ it makes $-2a$'s. For it takes away $2a$'s *more* than *all*, and therefore *leaves* $2a$'s *leſs* than *nothing*, or a *defect* of $2a$'s, like as he who hath 3 *pounds*, but owes 5 *pounds*, hath an Eſtate of -2 *pounds*, *that is*, is 2 *pounds* worſe than *nothing*.

But if the Species or Notes be *different*, and their proportion to one another *unknown* (whether the ſigns be *like* or *unlike*) they can be no *otherwiſe* connected but by the ſigns $+$ $-$ preſerving the Species *diſtinct*. Thus if to $3a$'s you add $2e$'s the Sum will be $3a+2e$, and in like manner, if to $3a$ you add $-2e$ the Aggregate or Sum will be $3a-2e$. Indeed if the proportion of a to e be *known* as that $2e$ is *equal* to $1a$, then would $3a+2e$ make $3a+1a$, *that is*, $4a$, but if *unknown* they cannot be otherwiſe added than thus, $3a+2e$.

CHAP. III.

Subſtraction *of* **Algebraick Integers.**

RULE.

Connect the Quantities given together as in Addition, only changing every ſign of the Quantity

tity to be ſubſtracted into its contrary, and the Sum thereof will be the difference or remainder ſought:

Examp:

From	$4a$	$5a$	$5a$	$c+b$
Take	a	$3a$	$-3a$	$a+b$
Remainder.	$4a-a$	$5a-3a$	$5a+3a$	$c+b-b-a$
that is,	$3a$	$2a$	$8a$	$c-a$

From	$c-b$	$b+c$
Subſtract	$a-b$	$-b-c$
Remainder	$c-b+b-a$	$2b+c+c$
that is,	$c-a$	$2b+2c$

The Reaſon hereof is likewiſe manifeſt from the Principles of common Arithmetick, according to which, if from 5 be taken 3, the *remainder* will be $5-3$ or 5 *wanting* 3, *that is*, 2 (For to take away 3, or ſubjoin the defect of 3 is all one) If from $+5$ you take -3, it makes $+5+3$, *that is*, $+8$; for to *take away* the defect of 3 is the ſame as to *ſupply* the 3 that were wanting, and ſignifies the taking away nothing *leſs* by 3, or *all but* 3, which is to add 3; for ſo much as is added to any Quantity, ſo much it wants of having nothing taken from it. If from -5 be taken -3 the *remainder* is $-5+3$, *that is*, -2; for now the defect is *leſs* by 3, which is the ſame as to *add* 3.

In like manner, if from $5a$ be taken $3e$, there remains $5a-3e$, if from $5a$ be taken $-3e$, it makes $5a+3e$, and if from $-5a$ be taken $+3e$, the *remainder* will be $-5a-3e$. For every where to *ſubſtract* a poſitive Quantity is the ſame as to *ſubjoin* a defect, and to *ſubſtract* a defect is the ſame as to *ſupply* or *add* ſo much.

Note, *That in Addition and Subſtraction it is all one in what order the ſeveral members connected by + or — do ſtand, ſo that each have its own ſign. As for inſtance,* $a-b+c$, *or* $a+c-b$, *or* $-b+a+c$, *or* $-b+c+a$ *are all of the ſame value.*

CHAP. IV.

Multiplication *of* Algebraic Integers.

RULE.

Conjoin the Quantities propoſed with the ſign ×, *or without, the ſign being underſtood.*

And here as in common Arithmetick *every* member of the *Multiplicand* is to be multiplyed by *each* member of the *Multiplier*; for the Multiplication of *all* the parts of the *one* by *all* the parts of the *other*, is the multiplication of the *whole* of the *one* by the *whole* of the *other*.

Examp. 1.

Multiply	a	$a+e$	$a-e$	$a+e+c$
by	e	b	b	z
Product	$a \times e$	$ab+eb$	$ab-eb$	$az+ez+cz$
or	ae			

Multiply

Multiply	$b+1$	$3a$	ae
by	a	$2a$	ae
Product	$ab+a$	$6aa$	$aaee$
or		$6a^2$	a^2e^2

Examp. 2.

Multiply	$a+e$	$a+e$	$a-e$
by	$a+e$	$a-e$	$a-e$
	$aa+ae$ $+ae+ee$	$aa+ae$ $-ae-ee$	$aa-ae$ $-ae+ee$
Product	$aa+2ae+ee$	$aa-ee$	$aa-2ae+ee$

In *Multiplication* that *like* ſigns give + and *unlike* — will appear from the *Notion* of *Multiplication*, which is to put the *Multiplicand* (whatever it be) ſo oft as there are *Units* or *parts* of an *Unit* in the *Multiplier*: For thus there may be a *double* defect or negative Quantity as well as a *double* poſitive Quantity, ſo that $-2a$ is as much the *double* of $-a$ as $+2a$ is the *double* of $+a$, which ſhews the *reaſon* why + by + makes + and — by + makes —, the *Multiplier* in *both* caſes, being a *poſitive* quantity, which puts the *negative* as well as the *poſitive* Quantity the *ſame* number of times. But if the *Multiplier* be a defect or negative Quantity, as -2 then inſtead of *putting*, it will ſignifie *taking away* the *Multiplicand* ſo many times: For as $+2$ implies *twice* putting, ſo -2 implies *twice* taking away the *Multiplicand*; (whether poſitive or negative): So that to multiply $+a$ by -2 is twice to take away a, and therefore doth produce a negative, *that is*, $-2a$, ſo + by — makes —: But to multiply $-a$ by -2 is *twice* to *take away* a defect or negative. Now to *take away* a defect, is the ſame as to *ſupply* it, and *twice* to *take away* or *ſupply* the defect of a, is the *ſame* as to *add* it *twice*, or to *put* $2a$, *that is*, *twice* to *take away* $-a$ is the *ſame* as *twice* to *add* $+a$: So that $-$ by $-$ (as well as $+$ by $+$) makes $+$. Thus to *multiply* $a-e$ by $a-e$ is to take $a-e$ ſo many times as a ſignifies (which makes $aa-ae$) *leſs* the *ſame* $a-e$ ſo many times as e ſignifies (which makes $ae-ee$) *that is*,

From $aa-ae\,(=:a-e:\times a)$
Subſtract $\underline{ae-ee\,(=:a-e:\times e)}$

And it *makes* $aa-2ae+ee$

And thus $-b$ *multiplied* by $-c$ (that is $0-b$ by $0-c$) will be $+bc$.

From $0\,(=:0-b:\times 0)$
Subſtract $\underline{0-bc\,(=:0-b:\times c)}$

and it will *make* $+bc$.

CHAP. V.

Diviſion of Algebraick Integers.

RULE.

PLace the Diviſor *under* the Dividend with a Line *between* them, and then if any Quantity be found to be a *common* Multiplier in *both* of them it is to be *put out* in both, and here alſo as in *Multiplication*) *like* ſigns give + and *unlike* —. Thus *dividing* by, or,

Applying To	$\frac{ae}{a}$	$\frac{ba+ca}{b-c}$	$\frac{6aa}{3a}$	that is $\frac{2\times3aa}{3a}$
Gives	e	a		$2a$

In theſe Examples it is apparent that if the *whole* Diviſor be found to be a *multiplier* in the *Dividend*, the Diviſion is then performed by *expunging* that *Multiplier*; (for *what is done by Multiplication is undone by Diviſion*, as if e be multiplied by a, and the *Product* or *Fact* divided by a the *Quotient* will be e); But if *not*, after *all expunctions* it will remain after the Form of a *Fraction*. Thus $\frac{ba}{ca} = \frac{b}{c}$. For $ba = \frac{cab}{c}\left(= \frac{b}{c}ca\right)$ wherefore dividing the equal Quantities, ba and $\frac{b}{c}ca$ by ca there will be $\frac{ba}{ca} = \frac{b}{c}$ (by *Ax.* 2.)

Note, *That it is all one in what order the Multiplications or Diviſions be made, ſo they be done ſucceſſively:* As,

$$ab = ba \quad \frac{ab}{c} = \frac{a}{c}b = \frac{b}{c}a$$

But ſometimes when the Quantities conſiſt of *many* Members, there may be a *common* Multiplier (or *common* meaſure of both parts, that *above*, and that *below* the line) which doth not *preſently* appear to the view, but may be found out after a *like* method, as in *common* Arithmetick. As thus, Firſt find *one* Member of the *Quotient*, then multiplying the *Diviſor* by it, ſubſtract this *Product* from the *Dividend*, and by the remainder ſeek for *another* member of the *Quotient*, and proceed as *before*, and ſo till the whole be *finiſhed*.

Thus if $aaa-eee$ be given to be divided by $a-e$, I enquire firſt what quantity multiplied by a (the *firſt* member of the *Diviſor*) will produce aaa, and find it to be aa, which multiplied by $a-e$ makes $aaa-aae$, and this ſubſtracted leaves $+aae-eee$. Then I enquire what multiplied into a will give aae, and I find it to be ae, which multiplied and ſubſtracted as *before*, leaves $aee-eee$, and in *like* manner I find the *third* and *laſt* member of the *Quotient*, which being ſo multiplied and ſubſtracted, leaves *nothing*. All which appears in the following Operation.

$$\begin{array}{rcl}
\textit{Diviſor} & \textit{Dividend} & \textit{Quotient.} \\
a-e) & aaa-eee & (aa,+ae,+ee \\
 & aaa-aae & \\
\hline
 & +aae-eee & \\
 & +aae-aee & \\
\hline
 & +aee-eee & \\
 & +aee-eee & \\
\hline
 & \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot &
\end{array}$$

In like manner, if the *Dividend* were divided by (or applied to $a+e$ the *Quotient* or Quantity thence *ariſing* would be $aa-ae+ee$.

The **Reaſon** of this Method of Operation will ſufficiently appear by conſidering the *proceſs* in *multiplying* $a-e$ the *Diviſor* by $aa+ae+ee$ the *Quotient*.

$$\begin{array}{lllll} \text{Multiply} & a-e & & & \\ \qquad\text{by} & aa+ae+ee & & & \\ \hline & aaa & -aae & & \\ & & +aae & -aee & \\ & & & +aee & -eee \\ \hline \text{which makes} & aaa & & & -eee \end{array}$$

And ſince *Diviſion* is but the *diſſolving* of a *Multiplication*, and what in *Multiplication* was the *Product*, is the *Dividend* in *Diviſion*, and what in *Multiplication* were the *two Factors* (*that is*, the *Multiplicand* and *Multiplier*) are in *Diviſion* the *Diviſor* and the *Quotient*, therefore theſe *two* muſt have *like* ſigns, if the *Dividend* be $+$, but *unlike* if that be $-$. So that if $+$ be divided by $+$, the *Quotient* muſt alſo be $+$, but if by $-$ the *Quotient* muſt alſo be $-$ (that theſe ſigns may be *like*) and if $-$ be divided by $+$ the *Quotient* muſt be $-$, if by $-$, the *Quotient* muſt be $+$, (that theſe ſigns may be *unlike*), *that is*, $+$ divided by $+$, or $-$ by $-$ makes $+$, but $+$ by $-$, or $-$ by $+$ makes $-$, thoſe being *like*, and theſe *unlike*.

CHAP. VI.

Reduction, Addition, Subſtraction, Multiplication, *and* Diviſion *of* Algebraick Fractions.

PROP. I.

TO Reduce a FRACTION into its leaſt terms.

RULE.

Divide the *Numerator* and *Denominator* by their *greateſt* common meaſure, and the *Quotients* will make an *equivalent* Fraction in the *leaſt* terms.

Examp.

$$\frac{a)ab}{a)ac}=\frac{(b}{(c}.\quad \frac{ab)abc}{ab)abd}=\frac{(c}{(d}.\quad \frac{4f)36ff\ldots\ldots}{4f)4bf+16df}=\frac{(9f}{(b+4d}$$

$$\frac{a+b)an+bn}{a+b)a^2+2ba+b^2}=\frac{(n}{(a+b}.\quad \frac{a+b)}{a+b)}\ \frac{aa+ab}{a+b}=\frac{(a}{(1}=a$$

The *Truth* of this Rule may be *demonſtrated* by the following Inſtances, *Viz.*

$\left\{\begin{array}{l}\textit{Dividend}\\ \textit{Diviſor}\end{array}\right.\ \frac{ab}{ac}=\frac{b}{c}=$ the Quotient. For herein by the nature of Diviſion, it appears that

$ab = \frac{b}{c} \times ac = \frac{bac}{c} = ba$ the *Dividend*, or $ab = \frac{b}{c} \times ac$ wherefore dividing the equal Quantities ab, $\frac{b}{c} \times ac$ by ac there will be (by *Ax.* 2.) $\frac{ab}{ac} = \frac{b}{c}$ *W.W.D.*

Or the Rule may be *demonſtrated* thus,

Since the *greater* the *Dividend* is, the *greater* is the *Quotient*, and the *greater* the *Diviſor* is, on the contrary the *leſs* is the *Quotient*, therefore if the *Dividend* and *Diviſor* be *multiplied* by the *ſame* Quantity, the *Quotient* will be ſtill the *ſame*. For then as by *multiplying* the Dividend, the Quotient will be encreaſed: So by *multiplying* the Diviſor, this Quotient will be as much *diminiſhed*, *that is*, it will become a Quotient equal to the *firſt*;

$$\text{Thus if } \left\{\begin{matrix}21 & \textit{Dividend}\\ \overline{7} & \textit{Diviſor}\end{matrix}\right\} \text{ be given.}$$

$$\text{Then will} \begin{cases} \frac{21\times3}{7} \text{ be } \textit{thrice} \sqsupset \frac{21}{7} \\ \frac{21\times3}{7\times3} \text{ be} \end{cases}$$

$$\left.\begin{matrix} \frac{21\times3}{7\times3} \text{ be} \\ \text{But (as above) } \frac{21}{7} \text{ is} \end{matrix}\right\} \textit{thrice} \sqsupset \frac{21\times3}{7}$$

$$\textit{Therefore} \quad \frac{21\times3}{7\times3} = \frac{21}{7} \; \textit{W. W. D.}$$

So that to bring the Fraction into the *leaſt* terms the *common* meaſure muſt be the *greateſt*; for the *greateſt* Diviſor makes the *leaſt* Quotient.

PROP.

PROP. II.

To Reduce an Integer into a Fraction of a given Denominator.

RULE.

Multiply the *Integer* by the given *Denominator*, and to the *Product* ſubſcribe the *ſame* Denominator.

Examp.

a Reduced to the *Denominator* f makes $\frac{fa}{f}=a$

$b+c$ reduced to the *Denominator* n makes $\frac{bn+cn}{n}=b+c$.

And thus an *Integer* may be expreſt *Fraction-wiſe*, by making an *Unit* its Denominator. As $a=\frac{a}{1}$.

PROP. III.

To Reduce an Integer with a Fraction into an improper Fraction.

RULE.

Multiply the *Integer* by the *Denominator* of the Fraction, and to the *Sum* of the Product added to the Numerator, ſubſcribe the *ſame* Denominator.

Examp.

$$b+\frac{c}{a} \text{ makes } \frac{ba+c}{a}=b+\frac{c}{a}.$$

PROP. IV.

To Reduce an Improper Fraction.

RULE.

Divide the *Numerator* by the *Denominator.*

Examp.

$$\frac{bdxcf}{d} \text{ makes } b+\frac{cf}{d}. \quad \frac{cg+gd}{c+d}=g \quad \frac{abd}{ab}=d.$$

PROP. V.

To Reduce Fractions of Divers Denominations into one Denomination.

RULE.

Multiply the *Numerator* of each Fraction into all the *Denominators* except it's *own*, and the ſeveral *Products* will be *new* Numerators, and then multiply *all* the Denominators together for a *common* Denominator.

Examp.

$\frac{ac}{b}$, $\frac{bd}{a}$ *reduced* make $\frac{aac}{ba}$ and $\frac{bbd}{ba}$

$\frac{c}{e}$, $\frac{f}{g}$, $\frac{adn}{h}$ *reduced* are $\frac{cgh}{egh}$, $\frac{feh}{egh}$, $\frac{adneg}{egh}$.

Demonſtration.

In the laſt Inſtance it is manifeſt that the Operation is in effect thus, $\frac{c \times g \times h}{e \times g \times h} \left(= \frac{cgh}{egh} = \frac{c}{e}\right)$ $\frac{f \times e \times h}{g \times e \times h} \left(= \frac{feh}{egh} = \frac{f}{g}\right)$. $\frac{adn \times e \times g}{h \times e \times g} \left(= \frac{adneg}{egh} = \frac{adn}{h}\right)$ and therefore Fractions of the *ſame* value, as appears by Demonſtration in *Prop.* 1.

PROP. VI.

To Add and Subſtract Fractions.

RULE.

If they be of *one* Denomination, Add or Subſtract their *Numerators*, and to the *Sum* or *Remainder* ſubſcribe the *common* Denominator, but if they have *divers* Denominators, reduce them (by *Prop.* 5.) to a *common* Denominator, and then Add or Subſtract as *before.*

Examp.

$\frac{a}{d}+\frac{b}{d}+\frac{c}{d}=\frac{a+b+c}{d}$ and $\frac{a}{d}+\frac{b}{d}-\frac{c}{d}=\frac{a+b-c}{d}$ and $\frac{a}{d}\pm\frac{c}{d}=\frac{a\pm c}{d}$.

$\frac{a}{b}, \frac{c}{d}$ *reduced* make $\frac{ad}{bd}, \frac{bc}{bd}$ and therefore $\frac{a}{b}\pm\frac{c}{d}=\frac{ad\pm bc}{bd}$.

PROP. VII.

To multiply Fractions.

RULE 1.

If *either* of the Quantities given be an *Integer*, and the other a *Fraction*, only *multiply* the *Numerator*, or (when it may be conveniently *done*) *divide* the *Denominator* by the *Integer*.

Examp.

$\frac{a}{b}\times d$ or $d\times\frac{a}{b}=\frac{ad}{b}$. $\frac{a}{bc}\times b=\frac{ba}{bc}=\frac{a}{c}=b)\frac{a}{bc}$.

RULE 2.

If *both* the Quantities given be *Fractions*, multiply the *Numerators* together for a *new* Numerator, and the *Denominators* for a *new* Denominator, and this *new* Fraction will be the *Product.*

Examp.

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ For $\frac{a}{b}\,c = \frac{ac}{b}$ and $d)\,\frac{ac}{b}\,(= \frac{ac}{db}$, becauſe $\frac{ac}{b}$ (the *Dividend*) =(the *Quotient*) $\frac{ac}{db} \times d$ (the *Diviſor*) $= \frac{dac}{db} =$ (by *Prop.* 1.) $\frac{ac}{b}$.

PROP. VIII.

To Divide Fractions.

RULE 1.

If a *Fraction* be given to be *divided* by an *Integer*, multiply the *Denominator*, (or if it can conveniently be done) Divide the *Numerator* by the *Integer.*

Examp.

$c\,)\,\frac{ac}{b}\,(\,\frac{c}{b}$. $\quad a)\,\frac{a}{b}\,(\,\frac{a}{ab} = \frac{1}{b}$.

Quotient Diviſor *Dividend*

$c)\,\frac{a}{b}\,(\,\frac{a}{cb}$. For $\frac{a}{cb} \times c = \frac{ac}{bc} = \frac{a}{b}$.

RULE

RULE 2.

If an *Integer* be given to be *divided* by a *Fraction* only *change* the terms of the *Diviſor*, and work as in *Multiplication.*

Examp.

$$\begin{array}{l} b \\ \frac{a}{b}\Big)\, c\, \Big(\frac{bc}{a}. \\ a \end{array}$$

For $\frac{a}{b} \times \frac{bc}{a} = \frac{abc}{ab} = c$ (by *Prop.* 1.)

$$\begin{array}{l} c \\ \frac{ab}{c}\Big)\, a\, \Big(\frac{c}{b} \\ ab \end{array} \quad \text{For } \frac{ab}{c} \times \frac{c}{b} = \frac{abc}{bc} = a.$$

RULE 3.

If a *Fraction* be given to be *divided* by a *Fraction*, work as in the *laſt foregoing* Rule, but if they have *equal Denominators*, Divide the *Numerator* of the *Dividend* by *that* of the *Diviſor*.

$$\frac{d}{c}\Big)\, \frac{a}{b}\, \Big(\frac{ac}{bd}.$$

For $\frac{ac}{bd} \times \frac{d}{c} = \frac{adc}{bdc} = \frac{a}{b}$ (by *Prop.* 1.)

$$\frac{a}{c}\Big)\, \frac{ab}{c}\, \Big(b \quad \text{and} \quad \begin{array}{l} c \\ \frac{d}{c}\Big)\, \frac{b}{c}\, \Big(\frac{bc}{dc} = \frac{b}{d}. \\ d \end{array}$$

A brief Synopſis *of the Operations in* Fractions.

$$\left.\begin{array}{ll} \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} & \textbf{Addition and Subſtr.} \\ \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} & \textbf{Multiplication.} \\ \frac{a}{b}\Big)\, \frac{c}{d} \Big(\frac{cb}{ad} & \textbf{Diviſion.} \end{array}\right\} \textit{of Fractions.}$$

CHAP. VII.

Of Proportions Arithmetical, Geometrical *and* Muſical.

NUmbers may be compar'd with one another two *wayes*.

1. They may be conſider'd with reſpect to their *difference*, *exceſs*, or *defect*, *that is*, by how much one is *greater* or *leſs* than another, which, it is evident, is ſhewn by *Subſtraction*.

2. They may be conſider'd with reſpect to their *ration*, or (as it is frequently called) *Proportion*, *that is*, how many times, or how much of a time or times one doth *contain*, or is *contained* in another, or (which is the ſame thing) how *much-fold* or what part or parts one is of another, and this is ſhewn by *Diviſion*. For if one Number be *divided* by another, the *Quotient* ſhews how oft the *Diviſor* is *contain'd* in the *Dividend*, or (if the Diviſor be *greater* than the Dividend) what part or

parts it is thereof. As 6) 12 (2 *that is*, the Quotient 2 ſhews that 6 is *contained* in 12, *twice*, but if 12) 6 ($\frac{6}{12}=\frac{1}{2}$ then the Quotient ($\frac{1}{2}$) expreſſes what *part* 6 is of 12, *that is*, one half part: Thus 12) 32 (2 $\frac{8}{12}$ = 2 $\frac{2}{3}$, *that is*, 32 *contains* 12 *two* times and two *thirds* of a time, but if 32) 12 ($\frac{12}{32}=\frac{3}{8}$, then the Quotient ($\frac{3}{8}$) ſhews that 12 is three *eighth* parts of 32; ſo that the *Quotient* is the *Denominator* of the Proportion of one Number to another: As if the Quotient be 2, the Proportion is *double*; if 3 *treble*, if 4 *quadruple*; if $\frac{1}{2}$ *ſubduple* or *one half*; if $\frac{1}{3}$ *ſubtriple* or *one third part*; if $\frac{3}{2}$ or 1 $\frac{1}{2}$ *ſeſquialter*, or the Proportion denominated by *one* and *an half*; if $\frac{4}{3}$ or 1 $\frac{1}{3}$ *ſeſquitertian* or *one* with a *third part*: And though the Quotient may be ſuch as that it cannot be expreſt either in *whole* Numbers or *Fractions*, yet *whatever* it is ſuppoſed to be, it *denominates*, or is the *Exponent* of the Proportion of one Number to another, it being according to the nature of Diviſion *ſuch* a Quantity as multiplying the Diviſor, the Product will *equal* the Dividend, *that is*, ſuch a Quantity as ſhews that ſo *much-fold* or *ſuch* a part or parts of the Diviſor make the Dividend, as that Quotient (whatever it be) *ſignifies*, which therefore *Denominates* their proportion to one another: So that *univerſally* the proportion of A to B is *denominated* or *expreſt by* $\frac{A}{B}$ *that is*, the Quotient of A *divided* by B, and therefore if r be put for the Proportion of one Quantity to another, then will $\frac{A}{B} = r$.

From this *twofold* compariſon of Numbers ariſes *two* kinds of Proportions commonly called *Arithmetical*, and *Geometrical* Proportions, in both which the Number compar'd is called the *Antecedent*, and that to which it is compar'd the *conſequent*.

1. **Arithmetical** Proportion is the *Equality* of differences, *that is*, when ſeveral Numbers have the *ſame* difference, and this proportion is *twofold*.

1. *Continual*, when of ſeveral Numbers the difference of the *firſt* and *ſecond* is the *ſame* with the difference of the *ſecond* and *third*, of the *third* and *fourth*, &c. Thus 1, 2, 3, 4, 5, *&c.* have the *ſame* difference, *viz.* 1. And thus 12, 9, 6, 3, have *likewiſe* the *ſame* difference, that is, 3. *This* proceeding of Numbers in a *continual* Arithmetical Proportion is called *Arithmetical Progreſſion*.

2. *Disjunct*, when there is the *ſame* difference between the *firſt* and *ſecond* that there is between the *third* and *fourth*, but *not* as between the *ſecond* and *third*. As for inſtance, 1 : 3 ∵ 7 : 9 are four *disjunct* Arithmetical Proportionals, the difference

of 1 and 3, and of 7 and 9, being 2, which is *not* the difference of 3 and 7, that being 4.

2. **Geometrical** Proportion is the *equality* of Rations, *that is*, when ſeveral Numbers do *contain* or are *contain'd* in one another the *ſame* Number of times, or *alike*, or which is the ſame thing being *divided* by one another have *equal* Quotients, which ſhew that they are *alike* contain'd in, or do contain one another, and *this* Proportion is likewiſe *twofold*.

1. *Continual*, when of ſeveral Numbers the *firſt* bears the *ſame* Proportion or Ration to the *ſecond* that the *ſecond* doth to the *third*, as the *third* doth to the *fourth*, &c. Thus (1. 2. 4. 8.) (2. 4. 8. 16) (24. 12. 6. 3.) are *continual* Geometrical Proportionals. In the firſt Inſtance as 1 is contain'd in 2 *twice*, ſo is 2 contain'd in 4, and 4 in 8 *twice*.

And *Thus*,

$$\left.\begin{array}{c}\frac{4}{2}\\[2ex]\frac{8}{4}\\[2ex]\frac{16}{8}\end{array}\right\}=2 \qquad \left.\begin{array}{c}\frac{24}{12}\\[2ex]\frac{12}{6}\\[2ex]\frac{6}{3}\end{array}\right\}=2$$

This proceeding of Numbers in a *continual* Geometrical Proportion is called *Geometrical Progreſſion*.

2. *Disjunct*; when the proportion of the *firſt* to the *ſecond* is the *ſame* with that of the *third* to the *fourth*, but *not* of the *ſecond* to the *third*. Thus 6 : 12 :: 9 : 18. are *disjunct* Geometrical Propor-

tionals, 6 being contain'd in 12 *as oft* as 9 is in 18, *that is*, twice, which is *not* the proportion of 12 to 9.

Muſical Proportion is when of four Numbers the *firſt* is to the *fourth* as the difference of the *firſt* and *ſecond* is to that of the *third* and *fourth*. As 5, 8, 12, 30 are *Muſical* Proportionals, becauſe $5 : 30 :: 8 - 5 : 30 - 12 :: 3 . 18$; Thus,

If a, m, n, e be *Muſical* Proportionals, then will $a : e :: m - a : e - n$.

CHAP. VIII.

Of the **Ways** *of* **Arguing** *in* **Geometrical Proportions.**

SInce in all Numbers the *Quotient* determines their *Ration* or *Proportion*; Therefore as the *Quotient* is, ſuch is the *Proportion*, *that is*, according as the Quotient is *greater* or *leſs*, ſo is the proportion *greater* or *leſs*; and if the Quotient be the *ſame*, or *equal*, the proportion will alſo be *equal*. So that where there is an *equality* of Quotients there is an *equality* of Rations, and where there is an *equality* of Rations, there will be an *equality* of Quotients. Wherefore though the Quotients of ſome Quantities *cannot* be exactly expreſt either in *whole* Numbers or *Fractions*, yet if they have *equal* Quotients, it is manifeſt that they are *proportional*, *that is*, bear equal proportion to one another, though *what* that proportion is be *inexpreſſible*, and if they have *equal* proportion, they muſt have *equal* Quotients, though theſe Quotients *cannot* be denoted by any *real* Numbers.

THEOREM 1.

If $\frac{A}{B}=\frac{a}{b}$ *that is*, if *that* Quotient be equal to *this*, then will $A:B::a:b$, or $B:A::b:a$, *that is*, either of theſe four will be *Proportionals*, by the *Definition* of Geometrical Proportion, *Chap.* 7. And ſince (by the Nature of Proportion, *Chap.* 7.) $\frac{A}{B}=r\left(=\frac{r}{1}\right)$ therefore $A:B::r:1$. Or if $A:B::a:b$: or $B:A::b:a$, then will $\frac{A}{B}=\frac{a}{b}$.

THEOREM 2.

If 2 Quantities be *multiplied* or *divided* by one and the *ſame*, ſuch *Products* and *Quotients* are *Proportionals* with them; becauſe the Quotient is ſtill the *ſame*. As $ma:mb::a:b$ becauſe $\frac{ma}{mb}=\frac{a}{b}$, by *Prop.* 1, *chap.* 6.

THEOREM 3.

In 4 *Disjunct* Proportionals the Product of the *firſt* and the *laſt* is equal to that of the *ſecond* and *third*, and in 3 *continual* Proportionals the Product of the *Extreme* terms is equal to the Square of that in the *middle*. As if $A:B::a:b$, then will

$Ab=Ba$: For ſince $\frac{A}{B}=\frac{a}{b}$ (by *Th.* 1.) therefore multiplying *both*, (*that is*, $\frac{A}{B}$ and $\frac{a}{b}$) by the *ſame* Quantity Bb (as $\frac{A}{B}\times Bb=Ab$, and $\frac{a}{b}\times Bb=Ba$) the Products will be $Ab=Ba$ (by *Ax.* 2. *chap.* 1.) Or *thus* ſince $\frac{A}{B}=\frac{a}{b}$ therefore (by the Nature of Diviſion) the *Dividend* $A=\frac{a}{b}\times B$ $(=\frac{Ba}{b})$ the *Diviſor* B multiply'd by $\frac{a}{b}=$ the *Quotient*, wherefore $\frac{Ba}{b}=A$, and conſequently $Ab=Ba$. And by the *ſame* reaſon, if $A.B.C$ ∺ (*that is* $A:B::B:C$) then will $AC=BB=B^2$.

THEOREM 4.

In 4 *Proportionals* ($A:B::a:b$) if the Product of the *ſecond* and the *third* be divided by the *firſt*, the Quot. will be equal to the *fourth*, as $\frac{Ba}{A}=b$.

which proves the truth of what is commonly called the *Golden Rule*, or *Rule* of *Proportion*, which is, of 4 Proportionals, 3 being *given*, to find the *fourth*, by multiplying the *third* by the *ſecond*, or the *ſecond* by the *third*, and dividing the Product by the *firſt*. For ſince (by *Th.* 3.) $Ba=Ab$, therefore dividing *both* by A, there will (by *Ax.* 2. *Chap.* 1.)

Chap. 1.) be $\frac{Ba}{A} = \frac{Ab}{A} (= b)$ and conſequently $\frac{Ba}{A} = b$. And by the *ſame* reaſon in 3 *continual* Proportionals $A.\ B.\ C \div$, the Square of the *Middle* term divided by one of the *Extremes*, gives the *other*. For ſince (by *Th.* 3.) $B^2 = AC$, therefore (dividing both by A) there will be $\frac{B^2}{A} = \frac{AC}{A} (= C)$. or (both by C) there will be $\frac{B^2}{C} = \frac{AC}{C} (= A)$ and conſequently $\frac{B^2}{C} = C$ or $\frac{B^2}{A} = A$. *W. W. D.*

THEOREM 5.

Theſe Quantities are in *continual* Proportion, $a.\ b.\ \frac{bb}{a}.\ \frac{bbb}{aa}.\ \frac{bbbb}{aaa}.$ &c. $\div$ each *foregoing* being in proportion to the *next following*, as a to b: For (*Th.* 4.) $a : b :: b : \frac{bb}{a}.\ a : b :: \frac{bb}{a} : \frac{bbb}{aa} \left(= a) \frac{bb}{a} \times b \right)$, and $a : b :: \frac{bbb}{aa} : \frac{bbbb}{aaa} \left(= a) \frac{bbb}{aa} \times b \right)$ &c.

THEOREM 6.

If 4 *Quantities* be *Proportional* ($A : B :: a : b$) they are alſo *Proportional* in *Alternation*, *Inverſion*, *Compoſition*, *Diviſion*, *Converſion* and *Mixtly*, and alſo in the *Alternations* and *Inverſions* of the *four* laſt.

If

(3c.) — If A.B :: a.b, or $\frac{A}{B} = \frac{a}{b}$ —

Then they will be so by

Alternation.. $A . a :: B . b$,

Invertion..... $B . A :: b . a$,

Composition. $A + B . B :: a+b . b$,

Dº. Altern̄... $A + B . a + b :: B . b$,

Dº. Invert̄... $B . A+B :: b . a+b$,

Division.. $A-B . B :: a-b . b$,

Dº. Altern̄... $A-B . a-b :: B . b$,

Dº. Invert̄.... $B . A-B :: b . a-b$,

Convertion.. $A . B \pm A :: a . b \pm a$,

Dº. Altern̄... $A . a :: B \pm A . b \pm a$;

Dº. Invert̄... $B \pm A . A :: b \pm a . a$,

Mixture. $A+B . A-B :: a+b . a-b$,

Dº. Altern̄.. $A+B . a+b :: A-B . a-b$,

Dº. Invert̄.. $A-B . A+B :: a-b . a+b$

For by Multiplying the Meanes & Extreames of each, we have $Ba = Ab$, ∴ by y^{e} 3^{d}. of this, are Proportionalls.

q.e.d,

THEOREM 7.

In *Proportional* Quantities how many ſoever they be, as one *Antecedent* is to its *Conſequent*, ſo is the *Sum* of all the *Antecedents* to the *Sum* of all the *Conſequents*. As if $A : a :: B : b :: C : c :: D : d$, &c. then will $A : a :: A+B+C+D$, &c. $: a+b+c+d$, &c.

For ſince $A : a :: B : b \therefore$ (by *Th.* 6.)
Compounding $A+B : a+b :: (B : b ::)\ C : c \therefore$
again $A+B+C : a+b+c :: (C : c ::)\ D : d$, &c.

THEOREM 8.

In *Proportionals* ($A.\ B.\ C.\ D$, &c. ∺, *that is*, $A : B :: B : C :: C : D$, &c.) continually *increaſing* (wherein every Quantity *next following* contains, or is greater than the *next aforegoing* ſo many times as B, the *ſecond*, contains, or is greater than A, the *firſt*,) putting a the *firſt* or *leaſt*, u the *laſt* or *greateſt*, and s the *Sum* of all (whereby $s-u=$ all the *Antecedents*, $s-a=$ all the *Conſequents*) then, by *Th.* 7. will,

$A : B$ (or $a : B$) $:: s-u : s-a \therefore$
(Th. 3.) $+as-aa=+Bs-Bu$ to both
Add $-as+Bu=-as+Bu \therefore$

(Ax. 1.) $+Bu-aa=+Bs-as$ both which
Divide by $B-a=B-a \therefore$ then will

(Ax. 2.) $\dfrac{Bu-aa}{B-a}=\dfrac{Bs-as}{B-a}=s$.

THEOREM

THEOREM 9.

In *Proportionals* ($A \,.\, B \,.\, C \,.\, D$, &c. *that is*, $A : B :: B : C :: C : D :$ &c.) continually *decreaſing* (wherein every Quantity *aforegoing* contains, or is greater than the *next following*, ſo many times as A, the *firſt*, contains or is greater than B the *ſecond*) putting u the *greateſt*, a the *leaſt*, and s the *Sum* of all. Then ſince

$A : B$, (or $r : 1$) $:: B : C :: C : D$, &c. $\therefore$ (Th. 6. *Dividing*, $A-B : B\,(r-1:1) :: B-C : C :: C-D : D \therefore$ (Th. 7.) $A-B : B\,(r-1:1) :: A-B, +B-C, +C-D$ $(=A-D=u-a) : B+C+D\,(=s-u)$ *wherefore* $A-B : B :: u-a : s-u$ or $r-1 : 1 :: u-a : s-u \therefore$

$$\frac{:u-a:\times B}{A-B}=s-u\text{, or }\frac{:u-a:\times 1=u-a}{r-1}=s-u$$

Or *Dividing*, $A-B : A\,(r-1:r) :: B-C : B :: C-D : C \therefore$ (Th: 7.) $A-B : A\,(r-1:r) :: A-B, +B, -C, +C-D$ $(=A-D=A-a=u-a) : A+B+C\,(=s-a) \therefore$ $A-B : A :: A-u : s-a$, or $r-1 : r :: u-a : s-a \therefore$

$$\frac{:A-a:\times A}{A-B}=s-a\text{, or }\frac{:u-a:\times r=ur-ar}{r-1}=s-a$$

$\therefore \dfrac{ur-ar}{r-1}=s-a$ to both which

Adding $+a$ $+a$ there will

be $\dfrac{ur-ar}{r-1}+a=s$, *that is*,

$$\frac{ur-ar+ar-a=ur-a}{r-1}=s.$$

CHAP. IX.

Compoſition *of* Proportions *or* Rations.

C*Ompoſition* of Proportions is *two-fold*, *viz.* The one by *Addition*, and the other by *Multiplication.*

1. *Compoſition* of Proportions by *Addition* (or *Addition* of *Proportions*) is performed by *Addition* of their *Exponents* to find the *Exponent* of the *Total*; As when the *treble* and the *double* make a *Quintuple* proportion, becauſe $3+2=5$. And thus the *Subſtraction* of Proportions is the *Subſtraction* of their *Exponents*, As when the *treble* wanting the *double* makes the *ſingle* Proportion (or that of *Equality*) becauſe $3-2=1$. Thus if the *Proportion* of *C* to *B*, or *C* to *D* be given to be *added* to or *ſubſtracted* from that of *A* to *B*, the *total* or *remainder* is found by *adding* or *ſubſtracting* their *Exponents*.

$$\frac{A}{B}+\frac{C}{B}=\frac{A+C}{B}\,.\,\frac{A}{B}-\frac{C}{B}=\frac{A-C}{B}\,.\,\frac{A}{B}+\frac{C}{D}=\frac{AD+BC}{BD}\,.$$

THEOREM 1.

If $\left.\begin{matrix}A\\C\end{matrix}\right\}:a::\left.\begin{matrix}B\\D\end{matrix}\right\}:b$ then is $A\pm C:a::B\pm D:b$.

For ſince $\frac{A}{a}=\frac{B}{b}$ and $\frac{C}{a}=\frac{D}{b}$, therefore (by *Ax.* 1.) $\frac{A}{a}\pm\frac{C}{a}=\frac{B}{b}\pm\frac{D}{b}$ *that is*, $\frac{A\pm C}{a}=\frac{B\pm D}{b}$:

Ergo $A\pm C:a::B\pm D:b$. *W. W. D.*

2. *Compoſition* of Proportions by Multiplication (or *Multiplication* of Proportions) is performed by *multiplying* their *Exponents*. As when the *double* of the *treble* makes the *Sextuple* proportion, becauſe $2\times3=6$. And thus *Diviſion* (or *diſſolution* of ſuch compoſition) is the *Dividing* the *Exponent* of the Dividend by *that* of the Diviſor. As when the *Subduple* of the *triple* makes the *Seſquialter*, becauſe $\frac{3}{2}=1\frac{1}{2}$. So if the *proportion* of *A* to *B* be given to be *multiplyed* or *divided* by *that* of *C* to *B*, or *C* to *D*, the *Product* or *Quotient* is found by *multiplying* or *dividing* the Exponents.

$$\frac{A}{B}\times\frac{C}{B}=\frac{AC}{B}.\quad \frac{A}{B}\times\frac{C}{D}=\frac{AC}{BD}.$$

$$\left.\frac{C}{B}\right)\frac{A}{B}\left(\frac{AB}{CB}=\frac{A}{C}.\quad \left.\frac{C}{D}\right)\frac{A}{B}\left(\frac{AD}{BC}.\right.\right.$$

Here *note that it is all one to Divide by that of* C *to* D, *or to multiply by its contrary, that of* D *to* C, *and therefore inſtead of the Diſſolution of a Proportion by Diviſion may be uſed a compoſition by Multiplication, that is, for inſtance inſtead of dividing by the double, to multiply by the Subduple, or inſtead of dividing the proportion of* A *to* B *by that of* C *to* D, *to compound with that of* D *to* C, *that is, to multiply by* $\frac{D}{C}$.

$$\frac{D}{C}\times\frac{A}{B}\left(=\frac{AD}{BC}\right)\text{ inſtead of }\left.\frac{C}{D}\right)\frac{A}{B}\left(=\frac{AD}{BC}\right).$$

THEOREM

THEOREM 2.

If between *any* termes propoſed (as *A*, *F*,) be interpoſed never ſo many *intermediate* terms (as *B*, *C*, D, *E*) whatever they be (whether all *greater* or all *leſs*, or *ſome* greater and *ſome* leſs than either *A* or *F*) the proportion of the *Extremes* is compounded of all the *Intermediates*, each with his next conſequent, *that is*, $\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} \times \frac{D}{E} \times \frac{E}{F} = \frac{A}{F}$, whereby it appears that the *intermediate* terms, being *above* and *below* the line, in *continual* Multiplication *deſtroy* themſelves, and the Product of the *whole* Multiplication is *only* $\frac{A}{F}$. *W. W. D.*

THEOREM 3.

If the proportion of theſe Quantities, *A*, *B*, *C*, *D*, &c. each to its conſequent be as that of *a*, *b*, *c*, *d*, &c. each to its conſequent; then is the *firſt* to the *laſt* of *thoſe*, as the *firſt* to the *laſt* of *theſe*, *that is*, if $\frac{A}{B} = \frac{a}{b}$, $\frac{B}{C} = \frac{b}{c}$, $\frac{C}{D} = \frac{c}{d}$, then is $\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$ *that is*, $\frac{A}{D}$ equal to $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$, *that is*, $\frac{a}{d}$: For the Factors *here* and *there* being *all* equal each to each *reſpectively*, the *Products* muſt alſo be equal, *that is*, $\frac{A}{D} = \frac{a}{d}$, and *therefore* $A : D :: a : d$. *W. W. D.*

THEOREM 4.

In $A, B, C,$ and $a, b, c,$ if $A : B :: b : c,$ and $B : C :: a : b,$ then will $A : C :: a : c,$ *that is*, if

$$\frac{A}{B}=\frac{b}{c}\text{ and }\frac{B}{C}=\frac{a}{b},$$

$$\text{then }\frac{A}{B}\times\frac{B}{C}\left(=\frac{A}{C}\right)=\frac{b}{c}\times\frac{a}{b}\left(=\frac{ab}{cb}=\frac{a}{c},\right)$$

that is, $\frac{A}{C}=\frac{a}{c}$, and *therefore* $A : C :: a : c$. W.W.D.

Every *Proportion* ariſing from *any* Number of *equal* proportions *compounded* by *Multiplication* is term'd ſuch a *Multiplicate* of *one* of theſe proportions as is correſpondent to the *Number* thereof.

Thus,

$$\text{If there be}\left\{\begin{array}{l}2\\3\\4\\\&c.\end{array}\right\}\textit{equal}\text{ Proportions,}\left\{\begin{array}{l}\text{the }\textit{compound}\\\text{proportion}\\\text{thence ariſ.}\\\text{is called the}\end{array}\right\}\left\{\begin{array}{l}\textit{Duplicate}\\\textit{Triplicate}\\\textit{Quadruplic.}\\\&c.\end{array}\right\}\textit{of one}\text{ of theſe Proportions.}$$

As the Proportion of A to B, whoſe Exponent is $\frac{A}{B}$.

$$\left.\begin{array}{l}\frac{A}{B}\times\frac{A}{B}\cdots\cdots\\[4pt]\frac{A}{B}\times\frac{A}{B}\times\frac{A}{B}\cdots\\[4pt]\frac{A}{B}\times\frac{A}{B}\times\frac{A}{B}\times\frac{A}{B}\\\&c.\end{array}\right\}\textit{that is}\left\{\begin{array}{l}\frac{A^2}{B^2}\\[4pt]\frac{A^3}{B^3}\\[4pt]\frac{A^4}{B^4}\end{array}\right\}\text{is the }\textit{Expon.}\text{ of the}\left\{\begin{array}{l}\textit{Duplicate}\\\textit{Triplicate}\\\textit{Quadrupl.}\\\&c.\end{array}\right\}\text{of }\frac{A}{B}$$

or if *a* be ſuppoſed to be the *Exponent* of the *Simple* or *Single* Proportion,

$$\text{then is } \left\{\begin{array}{l} a^2 \\ a^3 \\ a^4 \\ \text{\&c.}\end{array}\right\} \text{ the } \textit{Exponent} \text{ of its } \left\{\begin{array}{l}\textit{Duplicate}\\ \textit{Triplicate}\\ \textit{Quadruplic.}\\ \text{\&c.}\end{array}\right.$$

Thus, if *three* or *more* Quantities be in *continual* proportion (as 1, *a*, a^2, a^3, a^4, &c.) *that* of the *firſt* to the *third*, *fourth*, *fifth*, &c. (1 to a^2, a^3, a^4, *&c.*) is *Duplicate*, *Triplicate*, *Quadruplicate*, &c. of *that* of the *firſt* to the *ſecond* (1 to *a*). And by *Inverſion that* of the *third*, *fourth*, *fifth*, &c. is *Duplicate*, *Triplicate*, *Quadruplicate*, &c. of *that* of the *ſecond* to the *firſt*, *that is*, If $\frac{1}{a}$ or $\frac{a}{1}$ ſignifie the *ſimple* then will,

$$\left.\begin{array}{llr} \frac{1}{a^2} & \text{or } \frac{a^2}{1}, \textit{ that is}, & a^2 \\ \frac{1}{a^3} & \frac{a^3}{1} & a^3 \\ \frac{1}{a^4} & \frac{a^4}{1} & a^4 \\ \text{\&c.} & \text{\&c.} & \text{\&c.} \end{array}\right\} \text{ſignifie the} \left\{\begin{array}{l} \textit{Duplicate} \\ \textit{Triplicate} \\ \textit{Quadruplicate} \\ \text{\&c.} \end{array}\right\} \text{of ſuch } \textit{ſimple} \text{ Proportion.}$$

So that

$$\text{the} \left\{\begin{array}{l} \textit{Duplication} \\ \textit{Triplication} \\ \textit{Quadruplication} \\ \text{\&c.} \end{array}\right\} \begin{array}{l}\text{of a Pro-}\\ \text{portion}\\ \text{is the}\end{array} \left\{\begin{array}{l} \textit{Squaring} \\ \textit{Cubing} \\ \textit{Biquadrat.} \\ \text{\&c.} \end{array}\right\} \text{its } \textit{Expo.}$$

and

and the	*Subduplicat.*	of a Propor-tion is the *Extract.* the	*Square*	Root of
	Subtriplicat.		*Cube*	
	Subquadrupl.		*Biquadrat.*	
	&c.		&c.	

its *Exponent*, as will further appear by the following Chapter.

Proportions are *one* to *another* as are their *Exponents*, *thus* the *Double* is to the *Treble*, as 2 to 3, (and ſo of the reſt). But the *Duplicate* to the *Triplicate* is as the *Square* to the *Cube*, As if the *ſingle* Proportion be *double*, the *Duplicate* thereof (the *Double* of the *Double*, or 2×2) is to the *Triplicate* (the *Double* of the *Double* of the *Double* $2\times2\times2$) as the *Square* of 2 ($2\times2=4$) is to the *Cube* thereof ($2\times2\times2=8$). *Likewiſe* if the *ſingle* Proportion be *Subduple*, *that is*, one *half*, the *Duplicate* thereof (the *half* of the *half* or $\frac{1}{2}\times\frac{1}{2}$) is to the *Triplicate* (the *half* of the *half* of the *half*, or $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$) as the *Square* of $\frac{1}{2}$ $\left(\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\right)$ is to the *Cube* thereof $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}:\right)$ And ſo for other *Multiplicate* and *Submultiplicate* Proportions.

From what hath been ſaid, it is evident that the *Operations* about *Proportions*, or their *Exponents* are performed in *like* manner as in *Vulgar Fractions*.

CHAP.

CHAP. X.

Of the Nature and Compoſition of Squares, Cubes, and other Powers.

IF a given Quantity A be *continually* multiplied by its ſelf as AA or A^2, AAA or A^3, $AAAA$ or A^4, *&c.* then will A be the *Side*, *Root* or *firſt Power*, and A^2, A^3, A^4, *&c.* the *Square*, the *Cube*, the *Biquadrate*, *&c.* or the 2d, 3d, and 4th *Power* of A, *&c.*

And if 1 (One or Unit) be put for the *firſt* term, the Root A for the *ſecond*, and its *conſequent* Powers (in order) for the *third*, *fourth* terms, *&c.* they will be a *Rank* or *Progreſſion* of *Geometrical* Proportionals whoſe *common* Ration or Rate of Progreſſion is the *common* Multiplier A.

And ſince the *firſt* term 1 hath in it *no* dimenſion of the Root, the Root or *ſecond* term but *one*, the *third two*, *&c.* therefore if over 1 be placed a Cypher, and the Figures 1, 2, 3, *&c.* over the other terms in order according to the Powers of A, they will make a *Progreſſion* of *Arithmetical* Proportionals, and will be the *Exponents* or *Denominators* of the Powers or Degrees of A, ſhewing *how many* dimenſions of the Root are in each, or *how many* degrees it is from 1. As,

Expon. $0 \;.\; 1 \;.\; 2 \;.\; 3 \;.\; 4 \;.\; 5 \;.\; 6 \;.\; 7 \;.\; 8$, &c.
Powers $1 \;.\; A \;.\; A^2 .\; A^3 .\; A^4 .\; A^5 .\; A^6 .\; A^7 .\; A^8$, &c.

And *thus* it is evident from the Nature of Powers that they are all made by the *continual* Multiplication of the *Root*, according to which it's eaſie to conceive the *conſtruction* of the following *Table.*

A Table of the Powers of a Single Figure or Root.

a	a^2	a^3	a^4	a^5	a^6	a^7
1	1	1	1	1	1	1
2	4	8	16	32	64	128
3	9	27	81	243	729	2187
4	16	64	256	1024	4096	16384
5	25	125	625	3125	15625	78125
6	36	216	1296	7776	46656	279936
7	49	343	2401	16807	117649	823543
8	64	512	4096	32768	262144	2097152
9	81	729	6561	59049	531441	4782969

In the *firſt* Column of this Table are contain'd the 9 *Digits* as ſo many *Roots* or *Sides*, and in the *other* Columns are the *correſpondent* Powers to each Root as far as the *ſeventh* Power, and if there were occaſion might be continued farther, and by this Table may be diſcovered at *view* the *neareſt* Root (*Quadratic*, *Cubic*, &c.) of any Number whoſe Root requires not *more* than *one* Figure and the *reſpective* Power of ſuch Root.

But ſince the Root of any other Number muſt be found out by *parts* (as the Quotient is done in Diviſion) therefore it is conſider'd as conſiſting of 2 *parts*, $a+e$ (which is called a *Binomial* Root) *one part* whereof is ſuppoſed to be *already* known (or is to be found out by the foregoing Table) and the other *unknown* to be diſcover'd by the help of the following Table, which how it is made will be very evident by the *continual* Multiplication of the *Binomial* Root $a+e$, as followeth,

$$
\begin{array}{rll}
\text{Multiply} & a+e & \\
\text{by} & a+e & \\
\hline
 & a+ae & \\
 & \quad +ae+e^2 & \\
\hline
 & a^2+2ae+e^2 & \textit{Square} \\
 & a+e & \\
\hline
 & a^3+2a^2e+ae^2 & \\
 & \quad +a^2e+2ae^2+e^3 & \\
\hline
 & a^3+3a^2e+3ae^2+e^3 & \textit{Cube} \\
 & a+e & \\
\hline
 & a^4+3a^3e+3a^2e^2+ae^3 & \\
 & \quad +a^3e+3a^2e^2+3ae^3+e^4 & \\
\hline
 & a^4+4a^3e+6a^2e^2+4ae^3+e^4 & \textit{Biquadrate.} \\
 & \quad \&c. &
\end{array}
$$

By this Proceſs may eaſily be diſcerned the *Parts* of which each Power doth conſiſt, and how much is already *known*, and what remains to be *ſought out*.

The

A Table *of the* Powers *of a* Binomial *Root.*

1	Root	a	e								
2	a^2	$2ae$	e^2								
3	a^3	$3a^2e$	$3ae^2$	e^3							
4	a^4	$4a^3e$	$6a^2e^2$	$4ae^3$	e^4						
5	a^5	$5a^4e$	$10a^3e^2$	$10a^2e^3$	$5ae^4$	e^5					
6	a^6	$6a^5e$	$15a^4e^2$	$20a^3e^3$	$15a^2e^4$	$6ae^5$	e^6				
7	a^7	$7a^6e$	$21a^5e^2$	$35a^4e^3$	$35a^3e^4$	$21a^2e^5$	$7ae^6$	e^7			
8	a^8	$8a^7e$	$28a^6e^2$	$56a^5e^3$	$70a^4e^4$	$56a^3e^5$	$28a^2e^6$	$8ae^7$	a^8		
9	a^9	$9a^8e$	$36a^7e^2$	$84a^6e^3$	$126a^5e^4$	$126a^4e^5$	$84a^3e^6$	$36a^2e^7$	$9ae^8$	e^9	
10	a^{10}	$10a^9e$	$45a^8e^2$	$120a^7e^3$	$210a^6e^4$	$252a^5e^5$	$210a^4e^6$	$120a^3e^7$	$45a^2e^8$	$10ae^9$	e^{10}

In this Table it is manifeſt that in each power *a* and *e* have each of them an equal Number of Dimenſions, *that is*, ſo many as are expreſt by the *Index* and *Exponent* of the power, and that they be *continual Proportionals*, *viz*. thoſe of *a decreaſing*, but thoſe of *e Increaſing*, As in the 5th power there are 5 Dimenſions of *a*, and as many of *e*, *that is*,

a^5, a^4, a^3, a^2, a *Decreaſing*
e, e^2, e^3, e^4, e^5 *Increaſing*

And in *each* power the Number *prefixt* to every of the *intermediate* ſpecies is made up of the 2 Numbers *next* adjoyning in the *foregoing* power. As in the *5th*. Power 5 prefixt to a^4e is the *Sum* of 1 ſuppoſed to be prefixt to a^4 and 4 prefixt to a^3e in the *4th*. power, and ſo 10 in $10a^3e^2$ is the *Sum* of 6, and 4 in $4a^3e$, $6a^2e^2$ the *two next* adjoyning parts in the *foregoing* power.

So that by obſerving the *order* in which the parts of each power are *placed*, it will be eaſie (without *continual* Multiplication of the Root) to *begin* the Table, and *continue* it (ſo far as there may be occaſion) both as to the *Species* or *continual* proportionals, and to their prefixed Numbers.

Now as to the *Signs* + and —, whereby the parts of each power are *connected* to make up the *whole*. If in the Root the parts thereof be +, (as $a+e$) then will all the parts in any power thereof be likewiſe +, (becauſe + into + alwayes makes +) but if one of them be —. (as $a-e$) then in the power thereof, where the *Index* of $-e$ is an *even* Number, there the ſign is +, but where it is *odd*, there the Sign is —, (becauſe — into — makes +

and ſo doth any *even* Number of —, but + into —, or — into + makes —, and ſo doth any *odd* Number of —.) As for inſtance, If the Root be $a-e$, the *Cube* or *third* power thereof will be $a^3-3a^2e+3ae^2-e^3$, becauſe in the *ſecond* and *fourth* parts the *Indexes* for $-e$ are *odd*, *that is*, 1 and 3. But if in the Root *both* be —, then in the Cube *all* be —, (becauſe every where an *odd* Number of —) and ſo in every power whoſe *Index* is an *odd* Number. But in the *ſecond*, *fourth*, &c. power where the *Index* is an *even* Number, all be +, becauſe every where an *even* Number of —.

And as to the *place* or *value* of each Species that will eaſily appear by conſidering the Nature of each Dimenſion thereof. As for inſtance, ſuppoſing the Root to conſiſt of *two Figures*, as 68, then by conſidering each Figure *apart* by it ſelf (which is the moſt *commodious* way for practice), Let *a* repreſent 6, (*that is*, 60) and *e* 8, then muſt 36 ſtand in the *place* of *hundreds* (becauſe $60\times60=3600=a^2$) but 64 ($=8\times8=e^2$) muſt be in the *place* of *Units*, becauſe 8 is ſo many *Units* without any Cypher underſtood, then 48 ($=6\times8$) is really 480 ($=60\times8=ae$) and conſequently the double thereof is $960=2ae$, and therefore 96 ($=48\times2$) muſt be in the *place* of *Tens*, *that is*, a place *forwarder* than that *above*; So that the *whole Square* will ſtand thus.

$$\begin{array}{r|l}
68 & \textit{Root} \\
\hline
36\,.\,. & a^2 \\
\begin{array}{r}96\,.\\ 64\end{array} & \left.\begin{array}{l}2ae\\ e^2\end{array}\right\}\textit{Gnomon} \\
\hline
4624 & \textit{Square}
\end{array}$$

In like manner for the *Cube*, *a* repreſenting 6 (*that is* 60) and *e* 8, then muſt 216(=6x6x6) being *really* 216000 (=60x60x60) ſtand in the *place* of *Thouſands*, and 864 (being *indeed* 86400= 60x60x3x8=$3a^2e$) muſt be *one* place *forwarder*, which is that of *hundreds*, And 1152 (being *really* 11520=60x3x8x8=$3ae^2$) muſt be *another* place *forwarder*, *that is*, in that of *Tens*, but 512(=8x8x8=e^3) in the *Units* place, becauſe here is no Cypher wanting or underſtood, ſo that the *whole Cube* with its ſeveral parts will ſtand thus,

68	*Root*	
216...	a^3	
864..	$3a^2e$	*Gnomon*
1152.	$3ae^2$	
512	e^3	
314432	*Cube*	

And thus each part in every of the powers muſt have ſo many *places* left *vacant* as there are *Cyphers* underſtood to be *wanting*, and ſince each part hath *fewer* Cyphers by *one*, it muſt ſtand a place *forwarder* towards the *right* hand, than that *next* above it.

And the like proceſs is to be uſed in *other* Roots conſiſting of *three* or *more* Figures: Firſt, Taking the Figure next the *left* hand for *a*, and the next to it for *e*, then working according to the Table for that power, and all the parts being ſet in their due places and added together, the *Sum* is the *power* of theſe two Figures; then taking theſe two

Figures for *a*, and the next to them for *e*, and working as before, there will be the *power* of three Figures, and ſo on until the *power* of the *whole Root* be compleated.

57	209 *Root*
25..	$a^2=50000\times50000$
70.	$2ae=50000\times2\times7000$
49	$e^2=7000\times7000$
3249..	$a^2=57000\times57000$
228.	$2ae=57000\times2\times200$
4	$e^2=200\times200$
32718400..	$a^2=57200\times57200$
102960.	$2ae=57200\times2\times9$
81	$e^2=9\times9$
3272869681	*Square*

5 7	2 *Root*
125...	$a^3=500\times500\times500$
525..	$3a^2e=500\times500\times3\times70$
735.	$3ae^2=500\times3\times70\times70$
343	$e^3 = 70\times70\times70$
185193...	$a^3=570+570\times570$
19494..	$3a^2e=570\times570\times3\times2$
684.	$3ae^2=570\times3\times2\times2$
8	$E^3=2\times2\times2$
187149248	*Cube.*

And the *like* method is to be uſed in the *Compoſition* of *other* powers according to the *Nature* of each deſcribed in the Table *aforegoing*, and though the power of any known Root might be *otherwiſe* diſcover'd by the *common* way of *continual* Multiplication, yet is it *thus* done for the *clearer* Direction how to find out the *Root* of any power *required.*

CHAP. XI.

The Extraction *of the* Root *of a* Square, Cube, *or other* Power.

1. THE Method of *Extracting* or *finding out* the *Root* of a *given* Square or *ſecond* Power, is as followeth.

Suppoſe it be requir'd to *find out* the Square Root of 119025. Firſt *point* every *other* place, beginning with that of *Units.* Then ſeek (in the *firſt* Table, *chap.* 10.) the *greateſt* Square Number that may be *contain'd* in the *firſt* punctation (11) on the *left* hand, which in this caſe is 9, whoſe Root (3) ſet in the Quotient, and ſubſtract the Square (9) from 11, and to the Remainder (2) prefix (90) the Figures of the *next* Punctation, and there will be 290 (which is called the *Reſolvend.*)

Then *double* the Quotient 3, which makes 6, for a *Diviſor*, to the Reſolvend, leaving out its place of Units, *that is*, 29, in which inquire how oft 6? the anſwer is 4 times, (and 5 over, which with the next Figure (0) makes 50 a *ſufficient* Remainder for 16, the Square of 4) therefore place 4 in the Quotient, and prefix it to the Diviſor (6), and there will be 64, which multiply by 4, and ſubſtract the Product 256 (which is called the *Subtrahend*) from 290 the *Reſolvend*, and to the Remainder (34) prefix the *next* punctation (25), and there will be 3425 for a *new* Reſolvend.

Again *double* the Quotient 34, which makes 68, for a *Diviſor* to *this* Reſolvend, leaving out its place of *Units*, that is, 342, in which 68 being contained 5 times (with a ſufficient *Remainder* for 25, the *Square* of 5) place 5 in the Quotient, and prefix it to the Diviſor (68) and there will be 685, this multiplyed by 5, and the Product (3425) will be a *new* Subtrahend which being *equal* to the Reſolvend (3425) there will remain *nothing*. And thus the Operation is *finiſht*, and 345 found to be the *exact* Root of the Square *propoſed*.

$\cdot\ \ \cdot\ \ \cdot\ a\ e$

$119025(3\cdot4\cdot5$ *Root*

$a^2 = 3\times3 = 9 \ldots\ a\ \ e$

$2a+e$

$30\times2+4 = 64)290$ .. *Reſolvend.*

$:2a+e:\times e.$ $\quad = 2ae+e^2$

$64\times4 = 256$ .. *Subtrahend*

$2a+e$

$340\times2+5 = 685)3425$ *Reſolvend.*

$2ae+e:\times e.$ $\quad = 2ae+e^2$

$685\times5 = 3425$ *Subtrahend*

There is *another* Method of *Extraction* which differs but little from that *aforegoing*, and is performed in like manner as Diviſion; only putting the *firſt* Figure of the Root for the *firſt* Diviſor, and the *Double* of this Diviſor with the *ſecond* Figure of the Root annext; for the *ſecond* Diviſor, and to this, adding its *firſt* Figure towards the right hand, and to the Sum annexing the *third* Figure of the Root; for the *third* Diviſor, and ſo on for all the *remaining* Diviſors: And ſetting each Diviſor a place *forwarder* towards the right hand, and multiplying it by its reſpective Figure, and ſubſtracting (as in *Diviſion*,)

Thus in the *Examp.* annext, the *firſt* Diviſor 3, under 11, multiplied by 3 the *firſt* figure of the Root and ſubſtracted leaves 2.

```
2
119025(3
3
```

Then 6 the *Double* of the *laſt* Diviſor 3 (being ſet a place *forward* towards the right hand under 9) with 4 the *ſecond* Figure of the Root, makes 64, which multiplied (as in Diviſion) by 4, and ſubſtracted out of 290 leaves 34.

```
 234
119025(34
3..
 64
```

Laſtly, 4 the *firſt* Figure of 64 added to 64, makes 68, which (being ſet a place *forward* under 42) with 5 (the third and laſt Figure of the Root) makes 685, and this multiplied by 5, and ſubſtracted out of 3435 leaves *nothing*. And thus the Operation is *concluded*, and the Root the *ſame* as before.———

```
 234
119025(345
3....
 64..
  685
```

2. The Method of *Extracting* the *Cube Root*, or the *Root* of the *third power*, is as followeth.

Let it be required to find the *Cube Root* of 146363183.

Firſt, Set a point over the place of Units, and from thence over every *third* place, then (by the firſt Table, *chap.* 10.) find the *greateſt Cube* Number that may be contain'd in the *firſt* Punctation 146, and that (in this caſe) will be 125, the Root whereof 5 place in the Quotient, and ſubſtract the *Square* 125 from 146; and there will remain 21, to which prefix the *next* punctation 363, and there will be 21363 for a *Reſolvend.*

```
146363183(5
125...
-----
 21363
```

Then *Square* the Quotient, and *Triple* the Product, (as 5×5×3=75) for a *Diviſor* to the Reſolvend, leaving out the two firſt places on the right hand, *that is*, 213, in which the *Diviſor* is contain'd 2 times, (and a *ſufficient* remainder 6363 for 60 the *triple* of the Quotient 5 *multiplied* by 4 the *Square* of 2, and for 8 the *Cube* of 2, (each ſet a place *forwarder* towards the right hand) therefore place 2 in the Quotient, and ſet 150 (*that is* the 75 multiplied by 2, the Figure laſt plac't in the Quotient) under 213, and the other two Numbers 60 and 8 each a place forwarder, and theſe *three* Numbers added together will make

```
146363183(52
125......
-----
75)21363..
   -------
    150....
      60...
        8..
   -------
    15608..
   -------
     5755183
```

make a *Subtrahend* 15608, which ſubſtract from 21363 the Reſolvend above, and there will remain 5755, to which prefix the *next* Punctation 183, and there will be a *new* Reſolvend 5755183.

Again, *Square* the Quotient and *Triple* the Product (as 52x52x3=8112) for a *Diviſor* to the Reſolvend 5755183 leaving out the two firſt places, *that is*, 57551, in which the Diviſor 8112 is contain'd 7 times, and 767 over (to which being put the two next Figures 83, it will make 76783 a *ſufficient* Remainder for 7644 the *Triple* of the laſt Quotient 52 multiplied by 49 the *Square* of 7, and for 343 the *Cube* of 7, each ſet a place forwarder towards the right hand) therefore place 7 in the Quotient, and 56784 (*that is*, the Diviſor 8112 multiplied by 7 the Figure *laſt* plac't in the Quotient) under 57551, and the other two 7644, and 343 each a place *forwarder*, and theſe *three* Numbers added together make a *Subtrahend* 5755183 which being *equal* to the Reſolvend, there will remain nothing, and thus the whole Root appears to be *exactly* 527, and the whole Operation may be plainly repreſented to the *view* as followeth,

```
     .  .  .
    146363183(527
    125......
    ---
     31363...
     ----
     150.....
      60....
       8...
    -----
    15608...
    -----
8112)5755183
     -------
     56784..
      7644.
       343
     -------
     5755183
     -------
     .......
```

$\overset{a\;\;e}{}$

$14\dot{6}36\dot{3}18\dot{3}(\underbrace{5\;2}_{a}\;\underset{e}{7}$ *Root*

a^3 ———— $=125=5\times5\times5$

21363 *Reſolvend*

$3a^2$ ———— $=\;75=5\times5\times3$ *Diviſor*

$3a^2e$ ———— $=150\,..=5\times5\times3\times2$

$3ae^2$ ———— $=..60\,.=5\times3\times2\times2$

e^3 ———— $=...8=2\times2\times2$

$3a^2e+3ae^2+e^3=15608$ *Subtrahend*

5755183 *Reſolvend*

$3a^2$ ———— $=..8112=52\times52\times3$ *Diviſor*

$3a^2e$ ———— $=56784\,..=52\times52\times3\times7$

$3ae^2$ ———— $=..7644\,.=52\times3\times7\times7$

e^3 ———— $=....343=7\times7\times7$

$3a^2e+3ae^2+e^3=5755183$ *Subtrahend*

....

☞ Note, *That in all Extractions when a Diviſor cannot be found ſo oft as once in its Dividend, or if it could, and yet there ſhould ariſe a Subtrahend greater than the Reſolvend, in both theſe Caſes a Cypher muſt be put in the Quotient and prefixt to the laſt Diviſor* *for a new Diviſor, and the next Punctation added to the laſt Reſolvend for a new Reſolvend.*

```
    Examp. 1.                     Examp. 2.
                                     163
3 2                                1180079
 .  .  .  .                        . . . . .
7317025(2705                      8280818001(90999
2......                           9........
 47....                           180......
  540..                            1809....
   5405                             18189..
                                     181989
```

In the firſt Examp. the Diviſor 54 is not contain'd in 27 (part of the Reſolvend 270) ſo oft as *once*, and therefore a *Cypher* muſt be placed in the Quotient and prefixt to the Diviſor 54, and there will be a *new* Diviſor 540, and then 270 the *laſt* Reſolvend with the *next* Punctation 25 will make 27025 for a *new* Reſolvend: And in the *ſecond* Examp. though the Diviſor 18 is contain'd in 18, (part of the Reſolvend 180) *once*, yet if 1 be prefixt to 18, as 181, it will be a Subtrahend *greater* than its Reſolvend 180, and therefore in this caſe alſo a *Cypher* muſt be put in the Quotient, and then the *like* method of proceeding in the Operation muſt be uſed as in the *firſt* Examp.

The Method of *Extracting* the Roots of *other* Powers may be diſcover'd by obſerving the Conſtitution of Powers, (in *chap.* 10.) which ſhews what *manner* of Operation *each* Power requires.

If after the working as far as the Place of *Units* there ſhall yet be a *remainder*, the Operation may be *continued* by adding ſo many Punctations of *Cyphers*, (that is, ſo many *Two's* for the Square, *Three's*

Three's for the Cube, &c.) as the deſir'd exactneſs may require, and then proceeding as in *Integers*, and all the Figures in the Root beyond the place of *Units* will be in ſo many *Decimal* places.

The **Reaſon** of the aforegoing and other like Operations may ſufficiently appear as followeth.

7	8	0	3	*Root*	60	88	68	09	*Square*
49	..	..	..	*aa*	49	..	..	..	*aa*
					11	88	..	..	*Reſolvend*
					1	4 .	..	..	*2a*=70x2 *Diviſor*
11	2 . 64	.. ..	.. ..	*2ae* *ee*	11	2 . 64	.. ..	.. ..	*2ae* *ee*
60	84	..	..	*aa*	11	84	..	..	*Subtrahend*
						4	68	..	*Reſolvend*
						15	6 .	..	*2a*=780x2 *Diviſor* too big
60	84	00	..	*aa*		4	68	09	*Reſolvend*
						1	56	0 .	*2a*=7800x2 *Diviſor*
	4	68	0 . 9	*2ae* *ee*		4	68	0 . 9	*2ae*=2800x2x3 *ee*=3x3
60886809 the *Square* ſought						4	68	09	*Subtrahend*

5	4	3	*Root*	100	103	907	*Cube*
125	...	...	a^3	125	...	...	a^3
				35	103	...	*Reſolvend*
				7	5..	...	$3a^2=5\times5\times3$ *Divisor*
30 2	0.. 40. 64	... ...	$3a^2e$ $3ae^2$ e^3	30 2	0.. 40. 64	... ...	$3a^2e=5\times5\times3\times$ $3ae^2=5\times3\times4\times$ $e^3=4\times4\times4$
157	464	...	a^3	32	464	...	*Subtrahend*
				2	639	907	*Reſolvend*
					874	8..	$3a^2=54\times54\times$ *Divisor*
2	624 15	4.. 48. 27	$3a^2e$ $3ae^2$ e^3	2	624 15	4.. 48. 27	$3a^2e=54\times54\times$ $3ae^2=54\times3\times3$ $e^3=3\times3\times3$
160 103 907 the *Cube* ſought				2	639	907	*Subtrahend*
				.	...	...	

By *comparing* the Compoſition of the *Square* and *Cube* with the Reſolution thereof it will be eaſie to diſcern that the *Extraction* of the Square and Cube Roots is nothing elſe but *the undoing what is done by compoſition*; and ſo for other Powers.

And then as to the *Punctations* or *Pointings, viz.* that of the Square into *Two*'s, of the Cube into *Three*'s, and ſo in proportion for other Powers (that is, according to the *Index* thereof) beginning alwayes at the Units-place; and that then there muſt alwayes be as many *Figures* in the Root as there are *Points*, will appear from the Compoſition of the Power: As in the aforegoing inſtance 7803, if 7, which hath three places in it, (*that is,* 7000) be *ſquared*, it will then have 6 places after it, *&c.* if *Cubed*, 9 places after it, *&c.*

	7000	800
	7000	800
Square	49,000,000,	64,00,00
	7 000,	800
Cube	343,000,000,000	512,00,00,00

whereby it is evident that the Square of every Figure hath *twice* as many places after it as the Root hath, the Cube *three* times as many, *&c.* ſuitably for other Powers: And therefore the *Pointings* are ordered *accordingly*.

CHAP. XII.

Of the Reduction of Equations.

AN *Equation* is the mutual comparing of two *equal Quantities* differently expreſt (which are called the *two* parts thereof) and is uſually denoted with the ſign = between them and the *ſingle* Quantities of both parts are called the *termes* of the Equation. Thus $a=c+d^3+eg$ is an Equation in which the *ſingle* Quantity a, *one* part thereof, is equal to the compound Quantity $c+d^3+eg$, the *other* part, and the *termes* of both parts are the *ſingle* Quantities a, c, d^3, and eg.

An *Equation* is twofold, *viz.* } 1. *Pure* or *Simple*. 2. *Affected* or *Compound*.

A *pure* or *ſimple* Equation is that wherein the Quantity ſought (a) is expreſt by *one* power only, as by the 1ſt, 2d, 3d, 4th power, *&c.* Thus $2a+c=d+e$ and $ca^3=d+b$ are *ſimple* Equations.

An *Affected* or *Compound* Equation is that in which there are contained two or more different degrees or powers of the Quantity ſought (a). As in this Equation $a^3+2a^2+a^5c=cd$, there are three different powers of a, *viz.* a^3, a^2, a^5.

Equations are *reduced* as followeth.

1. By Addition.

This is performed by *adding* ſome Quantity in the Equation to both parts thereof. As for inſtance, ſuppoſing a to repreſent the *unknown* quantity.

To $a+b=c$	$a-b=c$	$c+a=0$	$c-a=0$
Add $-b$ $-b$	$+b$ $+c$	$-a$ $-a$	$+a$ $+a$
$a=c-b$	$a=c+b$	$c=0-a$	$c=0+a$
		$c=-a$	$c=a$

To

$$\begin{array}{l|l} \text{To } \quad 5a^2 - c = 2c - a^2 & a^3 - b - c^2 = f \\ \textit{Add } \; a^2 + c \quad\; c + a^2 & \qquad b + c^2 \quad b + c^2 \\ \hline \qquad 6a^2 = 3c & \qquad a^3 = f + b + c^2 \end{array}$$

$$\begin{array}{l|l} \text{To } \; a^2 - d^2 = c \quad + ba & ca - b = df - a^2 \\ \textit{Add } \; ba + d^2 \quad + d^2 - ba & a^2 + b \quad b + a^2 \\ \hline \qquad a^2 + ba = c + d^2 & ca + a^2 = df + b \end{array}$$

2. *By* **Subſtraction.**

This is done by *ſubſtracting* ſome Quantity in the Equation from *both* parts thereof. As in the following Examples.

$$\begin{array}{l|l} \text{From } \; a + b = c & 3a^2 + 2b^2 = c^2 + a^2 \\ \textit{Subſtract } \; + b \quad + b & \; a^2 + 2b^2 \quad 2b^2 + a^2 \\ \hline \qquad a = c - b & 2a^2 = c^2 - 2b^2 \end{array}$$

$$\begin{array}{l|l} \text{From } \; a^2 + c^2 + f = da + fg & a^2 + d + c = fa + bg \\ \textit{Subſtr.} \; da + c^2 + f \quad da + c^2 + f & fa + d + c \quad fa + d + c \\ \hline \qquad a^2 - da = fg - c^2 - f & a^2 - fa = bg - d - c \end{array}$$

The **Reaſon** of both theſe wayes of *Reduction* is evident (by *Ax.* 1.)

And from hence it is very apparent that any *term* in an Equation is transferr'd to the other ſide or part thereof with the *contrary* ſign, *that is*, if its ſign be — then with +, but if +, then with —; And this is called *Tranſpoſition.*

3. *By* **Multiplication.**

This is performed by *multiplying* both parts of the Equation by *ſome* Quantity therein. As for inſtance.

$$
\begin{array}{ll|l|l}
\textit{Multiply} & \frac{a}{c} = d & \frac{c^2}{a+b} = a & \frac{ca^2}{d} = \frac{g+f}{h} \\
\text{by} & c \qquad c & a+b \qquad a+b & dh \qquad dh \\
\hline
 & a = cd & c^2 = a^2 + ab & ca^2h = dg + df
\end{array}
$$

This ſhews the Method of reducing an Equation when one or both parts thereof are expreſt *Fraction-wiſe.*

4. *By* **Diviſion.**

This is done by *dividing* both parts of the Equation by *ſome* Quantity therein. As,

$$
\begin{array}{ll|l|l}
\textit{Divide} & a^2 = da & da - ca = b^2 & ba^2 + ca = bb \\
\text{by} & a \qquad a & d-c \qquad d-c & b \qquad b \\
\hline
 & a = d & a = \frac{b^2}{d-c} & a^2 + a\frac{c}{b} = b
\end{array}
$$

The **Reaſon** of both theſe laſt wayes of Reduction is evident, (by *Ax.* 2.)

5. *By* **Squaring, Cubing,** &c.

This (when one part only of the Equation hath a Radical Sign) is done by *rejecting* that ſign, and according to the nature thereof, *Squaring*, *Cubing*, &c. the other part; But if both parts of the Equation have like radical ſigns, then is it reduced, *only* by *rejecting* thoſe ſigns. As,

The *Squares* of	$\surd a = b\,d$	$\surd ba = c + d$
make	$a = b^2 d^2$	$ba = c^2 + 2cd + d^2$

The *Squares* of $\surd a = \surd^2 : \overline{c - d} :$

$$a = c - d$$

The *Cubes* of $\surd^3 : \overline{a + b} : = \surd^3 : c + d - b^2$

make $a + b = c + d - b^2$

5. *By* Extraction *of* Roots.

This is done by *extracting* like Roots of both parts of the Equation.

The *Squ. Roots* of	$a^2 = b^2$	$a^2 = c^2 + 2cd + d^2$	$a^2 = c$
make	$a = b$	$a = c + d$	$a = \surd c$

The *Cube R^ts*. of	$a^3 = b^3$	$a^3 = b^3 d^3$	$a^3 = b + c + d$
make	$a = b$	$a = b\,d$	$a = \surd^3 : \overline{b + c + d}$

The Reaſon of both theſe Reductions appears by *Ax.* 3, 4.

7. *By Converting* Proportions *into* Equations, *and* Equations *into* Proportions.

As if $d:b::d-a:a$	b,a,c ∺ *that is* $b:a::a:c$
Then will $da=bd-ba$	$a^2=bc$
by 1st hereof $da+ba=bd$	by 6th hereof $a=\sqrt{bc}$
5th $a=\frac{bd}{d+b}$	

If $3ba=cd$	$bd=da+ba$	$\frac{cd}{3b}=a$
Then will $3b:c::d:a$	$b:d+b::a:d$	$3b:c::d:a$

CHAP. XIII.

The Compoſition, *and* Preparation *of* Equations.

WHEN a Queſtion is propoſed, let the Quantity *ſought* be ſuppoſed to be *known*, and (the better to diſtinguiſh *known* from *unknown* Quantities) let it be repreſented by the *letter* (*a*) or ſome other *Vowel*, and the *given* Quantities by *Conſonants*. Then with the Quantities *thus expreſt* make your proceſs by *adding*, *ſubſtracting*, *multiplying*, *dividing*, *&c.* according as the nature of the Queſtion ſhall require, until you come to an Equation, which (if the Quantities therein *known* and *unknown* be variouſly intermixt) muſt be ſo reduced, as that the Quantity *ſought*, or ſome power thereof may make *one* part or ſide of the Equation and what is *known* the *other* ſide, *that is*, that it may be diſcover'd to be equal to ſomewhat *given* or *known*, in order to which Mr. *Oughtred* gives theſe *five* Rules following.

1. If the Quantity *ſought*, or any degree thereof be in *Fraction*, let the Equation be reduced by *Multiplication*, as directed in the foregoing Chapter. Thus if $a - c = \frac{a^2 + b^2}{d} + b + c$, then will $da - dc = a^2 + b^2 + db + dc$.

2. If *known* be intermingled with *unknown* Quantities let there be a *Tranſpoſition* of parts (as there is occaſion.) As if $da - dc = a^2 + b^2 + db + dc$, then by *tranſpoſition* of dc and a^2 there will be $da - a^2 = 2dc + db + b^2$.

3. If the *higheſt* Species or degree of the Quantity *ſought* be multiplied into any *given* Quantity, let all be *applied* to (or *divided* by) this *given* Quantity. As if $ba^2 + b^2a = z^3$, then *dividing* both parts by b, there will be $a^2 + ba = \frac{z^3}{b}$.

4. If it happen that all the *given Quant.* be multiplyed into any degree of that *ſought*, let there be a *depreſſion* of all (by *Diviſion*) to the *loweſt* degree thereof that may be. As if $a^4 + ba^3 = z^2a^2$, then *dividing* all by a^2, there will be $a^2 + ba = z^2$.

5. If one or more of them be a *Surd* Root, the Equation is to be purſued in the reſpective powers,

As if $\sqrt{ba} + b = c$, then by *tranſpoſition* $\sqrt{ba} = c - b$ and their *Squares* $ba = c^2 - 2cb + b^2$; and *dividing* both by b, there will be $a = \frac{c^2 - 2cb + b^2}{b}$. So if $\sqrt{:ba + ca:} - d = b$, then is $\sqrt{:ba + ca:} = b + d$, and $ba + ca = b^2 + 2bd + d^2$, and therefore $a = \frac{b^2 + 2bd + d^2}{b + c}$. Again, If $\sqrt{\frac{a}{3}} = \sqrt[3]{2a}$, that is, $\sqrt[6]{\frac{a^3}{27}} = \sqrt[6]{4a^2}$, then is $\frac{a^3}{27} = 4a^2$, and $a^3 = 108a^2$, and therefore $a = 108$.

Here it is ſuppoſed that the *known* Quantity hath always the ſign +, or is *affirmative*. And therefore when it hath the ſign −, it is made + or *affirmative* by *tranſpoſition*. As if $a^2 - ba = -z^2$ by *tranſpoſition* with contrary ſigns, there will be $z^2 = -a^2 + ba$, or $z^2 = ba - a^2$.

Note, That the Powers of Quantities may be expreſt in the following manner.

$$\text{Thus}\left\{\begin{array}{l} Q:a+b: \\ C:a+b: \\ :a+b:(2) \\ \\ :a+b:(3) \\ \\ :a+b:(4) \\ \quad \&c. \end{array}\right\}\text{ſignifies the}\left\{\begin{array}{l} \textit{Square} \\ \textit{Cube} \\ \textit{Square or} \\ 2^{d}\,\textit{Power} \\ \textit{Cube, or } 3^{d} \\ \textit{Power} \\ 4^{th}\,\textit{Power} \\ \quad \&c. \end{array}\right\}\text{of } a+b$$

Obſerve alſo, that according to the Method of Managing Equations (in *Chap.* next following) by keeping an account of the Proceſs in the *Margin*, or firſt *Column* next the left hand, each Figure in the ſecond *Column* diſtinguiſhes the Equation ſtanding againſt it in the third *Column*. As 10 againſt $2a = z + x$ ſhews this is the tenth Equation, and $1 + 2$ in the firſt Column ſignifies, that if the terms of the firſt Equation be added to thoſe of the ſecond, there will thence ariſe *another* Equation, $2a = z + x$; but where a figure in the firſt Column hath a *point* prefixt, there it is conſidered as an *abſolute* Number, *thus* in the firſt Column againſt the eleventh Equation, $10 \div 2.$ ſignifies the tenth Equation divided by the *abſolute* Number 2, and where there are Figures in the firſt Column without any ſign, they denote the *comparing* of Equations, *as* 88, 90 againſt the 91ſt Equation ſignifie that $z^3 = z + 3za$ ariſes by *comparing* the 88th Equation with the 90th.

CHAP. XIV.

Of **Compendious Notes** *or* **Characters,** *with the uſe thereof.*

SUCH *Compendious Notes* or *Characters* as Mr. *Oughtred* makes uſe of in his *Clavis Mathematica* are ſo great helps to the memory or fancy, that thereby may be diſcover'd and apprehended (with much more eaſe than can be done by words) ſeveral very *uſeful* Equations which do either appear even upon the *firſt* view of the terms, or may be very *readily* form'd by the *common* Operations of *Addition*, *Subſtraction*, *Multiplication*, &c. *Thus* ſuppoſing there be two Quantities, the greater whereof is *a*, the leſſer *e*, then,

Putting

	1	$z = a + e$
	2	$x = a - e$
	3	$\text{æ} = ae$
	4	$Z = a^2 + e^2$
	5	$X = a^2 - e^2$
	6	$\text{æ}^2 = a^2e^2$
	7	$\mathcal{Z} = a^3 + e^3$
	8	$\mathcal{X} = a^3 - e^3$
	9	$\text{æ}^3 = a^3e^3$

1 + 2	10	$2a = z + x$
10 ÷ 2.	11	$a = \frac{1}{2}z + \frac{1}{2}x$
1 — 2	12	$2e = z - x$
12 ÷ 2.	13	$e = \frac{1}{2}z - \frac{1}{2}x$
1 × 2	14	$zx = a^2 - e^2 = X$
Q : 1	15	$z^2 = a^2 + 2ae + e^2$ $= Z + 2\text{æ}$
Q : 2	16	$x^2 = a^2 - 2ae + e^2$ $= Z - 2\text{æ}$
15 + 16	17	$z^2 + x^2 = 2a^2 + 2e^2$ $= 2Z$
15 — 16	18	$z^2 - x^2 = 4\text{æ}$
18 ÷ 4.	19	$\text{æ} = \frac{1}{4}z^2 - \frac{1}{4}x^2$ $= Q : \frac{1}{2}z : - Q : \frac{1}{2}x$

1 — a	20	$e = z - a$
10 — z	21	$x = 2a - z$
20 × a	22	$\text{æ} = za - a^2$
a^2 + Q : 20	23	$Z = z^2 - 2za + 2a^2$ $= 2\,Q : \frac{1}{2}z : + 2\,Q : \frac{1}{2}z - a :$
a^2 — Q : 20	24	$X = 2za - z^2$

1 — e

$1 - e$	25	$a = \zeta - e$
$25 - e$	26	$x = \zeta - 2e$
$25 \times e$	27	$æ = \zeta e - e^2$
$Q : 25 : + e^2$	28	$z = \zeta^2 - 2\zeta e + 2e^2$
		$= 2\,Q : \frac{1}{2}\zeta : + 2\,Q : \frac{1}{2}\zeta - e :$
$Q : 25 : - e^2$	29	$x = \zeta^2 - 2\zeta e$
$2 + e$	30	$a = e + x$
$30 + e$	31	$\zeta = 2e + x$
$30 \times e$	32	$æ = e^2 + xe$
$Q : 30 : + e^2$	33	$z = 2e^2 + 2xe + x^3$
$Q : 30 : - e^2$	34	$x = 2xe + x^2$
$30 - x$	35	$e = a - x$
$35 + a$	36	$\zeta = 2a - x$
$35 \times a$	37	$æ = a^2 - xa$
$a^2 + Q : 35$	38	$z = 2a^2 - 2xa + x^2$
$a^2 - Q : 35$	39	$x = 2xa - x^2$
$3 \div a$	40	$e = \frac{æ}{a}$
$a + 40$	41	$\zeta = a + \frac{æ}{a} = \frac{a^2 + æ}{a}$
$a - 40$	42	$x = a - \frac{æ}{a} = \frac{a^2 - æ}{a}$
$a^2 + Q : 40$	43	$z = a^2 + \frac{æ^2}{a^2}$
		$= \frac{a^4 + æ^2}{a^2}$
$a^2 - Q : 40$	44	$x = a^2 - \frac{æ^2}{a^2}$
		$= \frac{a^4 - æ^2}{a^2}$

$3 \div e$

$3 \div e$	45	$a = \frac{a}{e}$
$45 + e$	46	$z = \frac{a}{e} + e$
		$= \frac{a + e^2}{e}$
$45 - e$	47	$x = \frac{a}{e} - e$
		$= \frac{a - e^2}{e}$
$Q : 45 : + e^2$	48	$z = \frac{a^2}{e^2} + e^2$
		$= \frac{a^2 + e^4}{e^2}$
$Q : 45 : - e^2$	49	$x = \frac{a^2}{e^2} - e^2$
		$= \frac{a^2 - e^4}{e^2}$
		If $r . s :: a . e$
then will	50	$re = sa$
$50 \div e$	51	$e = \frac{sa}{r}$
$a + 51$	52	$z = a + \frac{sa}{r}$
		$= \frac{ra + sa}{r}$
$a - 51$	53	$x = a - \frac{sa}{r}$
		$= \frac{ra - sa}{r}$

$51 \times a$	54	$ae = \frac{sa^2}{r}$
$a^2 + Q: 51$	55	$z = a^2 + \frac{s^2a^2}{r^2}$
		$= \frac{r^2a^2 + s^2a^2}{r^2}$
$a^2 - Q: 51$	56	$x = a^2 - \frac{s^2a^2}{r^2}$
		$= \frac{r^2a^2 - s^2a^2}{r^2}$
$50 \div s$	57	$a = \frac{re}{s}$
$57 + e$	58	$z = \frac{re}{s} + e$
		$= \frac{re + se}{s}$
$57 - e$	59	$x = \frac{re}{s} - e$
		$= \frac{re - se}{s}$
$57 \times e$	60	$ae = \frac{re^2}{s}$
$Q: 57 : + e^2$	61	$z = \frac{r^2e^2}{s^2} + e^2$
		$= \frac{r^2e^2 + s^2e^2}{s^2}$
$Q: 57 - e^2$	62	$x = \frac{r^2e^2}{s^2} - e^2$
		$= \frac{r^2e^2 - s^2e^2}{s^2}$

$4 - e^2$	63	$a^2 = Z - e^2$
$5 + e^2$	64	$a^2 = X + e^2$
$4 - a^2$	65	$e^2 = Z - a^2$
$64 - X$	66	$e^2 = a^2 - X$
$11 \times a$	67	$a^2 = \frac{1}{2} za + \frac{1}{2} xa$
$11 \times e$	68	$ae = \frac{1}{2} ze + \frac{1}{2} xe$
$13 \times e$	69	$e^2 = \frac{1}{2} ze - \frac{1}{2} xe$
$13 \times a$	70	$ae = \frac{1}{2} za - \frac{1}{2} xa$
$15 - 2ae$	71	$Z = z^2 - 2ae$
$15 - Z$	72	$2ae = z^2 - Z$
$72 \div 2.$	73	$ae = \frac{1}{2} z^2 - \frac{1}{2} Z$
$16 + 2ae$	74	$Z = x^2 + 2ae$
$74 - x^2$	75	$2ae = Z - x^2$
$75 \div 2.$	76	$ae = \frac{1}{2} Z - \frac{1}{2} x^2$
$17 \div 2$	77	$Z = \frac{1}{2} z^2 + \frac{1}{2} x^2$
		$= 2 Q : \frac{1}{2} z : + 2 Q : \frac{1}{2} x :$
$20 \times e$	78	$e^2 = ze - ae = Q : z - a :$
$25 \times a$	79	$a^2 = za - ae = Q : z - e :$
$30 \times a$	80	$a^2 = ae + xa = Q : e + x :$
$35 \times e$	81	$e^2 = ae - xe = Q : a - x :$
$80 + 78$	82	$Z = ze + xa$
$79 + 81$	83	$Z = za - xe$
$80 - 81$	84	$X = xa + xe$
$79 - 78$	85	$X = za - ze$
$79 - 81$	86	$X = za + xe - 2ae$
$80 - 78$	87	$X = xa + 2ae - ze$
C: 1	88	$z^3 = a^3 + 3a^2e + 3ae^2 + e^3$
		$= Z + 3a^2e + 3ae^2$
1×3	89	$zae = a^2e + ae^2$
$89 \times 3.$	90	$3zae = 3a^2e + 3ae^2$
88, 90	91	$z^3 = Z + 3zae$
C: 2	92	$x^3 = a^3 - 3a^2e + 3ae^2 - e^3$
		$X - 3a^2e + 3ae^2$
2×3	93	$xae = a^2e - ae^2$

$93 \times 3.$	94	$3xa = 3a^2e - 3ae^2$
92, 94	95	$x^3 = x - 3xe$
1×4	96	$zz = a^3 + a^2e + ae^2 + e^3$
		$= z + a^2e + ae^2$
89, 96	97	$zz = z + za$
2×4	98	$zx = a^3 - a^2e + ae^2 - e^3$
		$x - a^2e + ae^2$
93, 98	99	$zx = x - xa$
1×5	100	$xz = a^3 + a^2e - ae^2 - e^3$
		$= x + a^2e - ae^2$
93, 100	101	$xz = x + xa$
2×5	102	$xx = a^3 - a^2e - ae^2 + e^3$
		$= z - a^2e - ae^2$
89, 102	103	$xx = z - za$
$97 + 103$	104	$2z = zz + xx$
$97 - 103$	105	$2za = zz - xx$
$99 + 101$	106	$2x = xz + zx$
$101 - 99$	107	$2xa = xz - zx$
$4 + 5$	108	$2a^2 = z + x$
$4 - 5$	109	$2e^2 = z - x$
$108 \div 2.$	110	$a^2 = \frac{z + x}{2}$
$109 \div 2.$	111	$e^2 = \frac{z - x}{2}$
Q: 4	112	$z^2 = a^4 + 2a^2e^2 + e^4$
Q: 5	113	$x^2 = a^4 - 2a^2e^2 + e^4$
$112 - 113$	114	$4a^2e^2 = 4a^2 = z^2 - x^2$
$114 + x^2$	115	$z^2 = 4a^2 + x^2$
$115 - 4a^2$	116	$x^2 = z^2 - 4a$
$\sqrt{}\ 116$	117	$x = \sqrt{} : z^2 - 4a :$
110, 111,	118	$\sqrt{} : \frac{z + (x =) \sqrt{} : z^2 - 4a}{2} :$
117		$\pm \sqrt{} : \frac{z - (x =) \sqrt{} : z^2 - 4a}{2} :$
		$= \sqrt{a^2} \pm e^2 = \sqrt{a} \pm e.$

Out of the *aforegoing* Equations may be collected various *Designations* of the *same* Quantity; As followeth.

$$a = \begin{cases} & \text{Eq.} \\ \frac{1}{2}z + \frac{1}{2}x & 11 \\ z - e & 25 \\ e + x & 30 \\ \frac{\text{æ}}{e} & 45 \\ \frac{re}{s} & 57 \end{cases} \qquad e^2 = \begin{cases} & \text{Eq.} \\ Z - a^2 & 65 \\ a^2 - x & 66 \\ \frac{1}{2}ze - \frac{1}{2}xe & 69 \\ ze - ae & 78 \\ ae - xe & 81 \\ Q{:}\, z - a{:} & 78 \\ Q{:}\, a - x & 81 \end{cases}$$

$$e = \begin{cases} \frac{1}{2}z - \frac{1}{2}x & 13 \\ z - a & 20 \\ a - x & 35 \\ \frac{\text{æ}}{a} & 40 \\ \frac{sa}{r} & 51 \end{cases}$$

$$z = \begin{cases} a + e & 1 \\ 2e + x & 31 \\ 2a - x & 36 \\ \frac{a^2 + \text{æ}}{a} & 41 \\ \frac{\text{æ} + e^2}{e} & 46 \\ \frac{ra + sa}{r} & 52 \\ \frac{re + se}{s} & 58 \end{cases} \qquad \text{æ} = \begin{cases} ae & 3 \\ \frac{1}{4}z^2 - \frac{1}{4}x^2 & 19 \\ Q{:}\, \frac{1}{2}z{:} - Q{:}\, \frac{1}{2}x & 19 \\ za - a^2 & 22 \\ ze - e^2 & 27 \\ e^2 + xe & 32 \\ a^2 - xa & 37 \\ \frac{sa^2}{r} & 54 \\ \frac{re^2}{s} & 60 \\ \frac{1}{2}ze + \frac{1}{2}xe & 68 \\ \frac{1}{2}za - \frac{1}{2}xa & 70 \\ \frac{1}{2}z^2 - Z & 73 \\ \frac{1}{2}Z - \frac{1}{2}x^2 & 76 \end{cases}$$

$$x = \left\{\begin{array}{l|c} a - e & 2 \\ 2a - z & 21 \\ z - 2e & 26 \\ \dfrac{a^2 - 2a}{a} & 42 \\ \dfrac{a - e^2}{e} & 47 \\ \dfrac{ra - sa}{r} & 53 \\ \dfrac{re - se}{s} & 59 \end{array}\right.$$

$$z = \left\{\begin{array}{l|c} a^2 + e & 4 \\ 2Q{:}\tfrac{1}{2}z + 2Q{:}\tfrac{1}{2}z - a{:} & 23 \\ 2Q{:}\tfrac{1}{2}z + 2Q{:}\tfrac{1}{2}z - e{:} & 28 \\ z^2 - 2a & 71 \\ x^2 + 2a & 74 \\ \tfrac{1}{2}z^2 + \tfrac{1}{2}x^2 & 77 \\ 2Q{:}\tfrac{1}{2}z{:} + 2Q{:}\tfrac{1}{2}x{:} & 77 \\ ze + xa & 82 \\ za - xe & 83 \end{array}\right.$$

$$a^2 = \left\{\begin{array}{l|c} z - e^2 & 63 \\ x + e^2 & 64 \\ \tfrac{1}{2}za + \tfrac{1}{2}xa & 67 \\ za - ae & 79 \\ ae + xa & 80 \\ Q{:}z - e{:} & 79 \\ Q{:}e + x & 80 \end{array}\right.$$

$$x = \left\{\begin{array}{l|c} a^2 - e^2 & 5 \\ zx & 14 \\ 2za - z^2 & 24 \\ z^2 - 2ze & 29 \\ 2xe + x^2 & 34 \\ 2xa - x^2 & 39 \\ xa + xe & 84 \\ za - ze & 85 \\ za + xe - 2a & 86 \\ xa + 2ae - ze & 87 \end{array}\right.$$

CHAP. XV.

Of the Nature, Derivation *and* Reſolution *of* Affected *or* Compound Quadratick Equations.

AN *Affected-Quadratic Equation* is that wherein there are only *two* different powers of the Quantity *ſought*, one whereof is a *Square*, whoſe ſide or Square Root is the *other*, *that is*, they are ſuch that the Index or Exponent of the *one* is *double* the Index of the *other*. As $a^2+ba=c$, or $a^6+da^3=b$, in *one* Equation the Index of a^2 is 2, which is the *double* of 1 the Index of a (in ba), and in the *other* the Index of a^6 is 6, which is the double of 3 the Index of a^3 (in da^3).

The *given* Quantities which are *multiplied* by, or do *multiply* ſome power of the Quantity *ſought* are commonly called *Coefficients*. As in $a^2+ba=c$ the *given* Quantity b is the *Coefficient*, and in $ma^2+\frac{c+d}{b}a=f$, the Coefficients are m, and $\frac{c+d}{b}$. And when the lower power is *ſingle* without any Quantity drawn into it, then it is always ſuppoſed to have 1 (or an *Unit*) for its *coefficient*, As in this Equation, $a^2+a=d$, the *coefficient* of a is 1.

Theſe Equations are uſually ſuppoſed to conſiſt of *three* terms, *Viz.*

1. The *higheſt* power of the Quant. *ſought* with or without a *coefficient*.	which is called the	*Higheſt* term.	As in $a^2+ca=b$ or in $ma^2+ca=b$	a^2
2. The *lower* power with its *coefficient*.		*Middle* term.		ca
3. The *term* to which the other two are *equal*		*Abſolute* Quantity *given*.		b

And *theſe* are the Equations which have (as Mr. *Oughtred* expreſſeth it) 3 *Species equally aſcending in the Scale, that is*, whoſe *Indexes* of the Quantity *ſought* are in *Arithmetical Progreſſion.*

$$\text{As}\left\{\begin{matrix} a^2, & a, & 1 \\ a^4, & a^2, & 1 \\ a^6, & a^3, & 1 \\ & \text{\&c.} & \end{matrix}\right\}\text{whoſe } \textit{Indexes} \text{ are}\left\{\begin{matrix} 2, & 1, & 0 \\ 4, & 2, & 0 \\ 6, & 3, & 0 \\ & \text{\&c.} & \end{matrix}\right.$$

And though in *theſe* Equations he reckons but *three* Species, yet they may be ſuch as have *more* than *three* terms or members, as the *following* Equation,

$$ma^2 + a^2 + ba - ca = e^2 + b^2 - mn$$

in which there are *ſeven* terms, but the *Species* are reputed to be no *more* than *three* (according to the *Indexes* of the Root or Quantity *ſought*, as here 2, 1, 0) *Viz.*

I. The *Higheſt* term	denominated by	The *Square* (a^2), As $ma^2 + a^2$ (in which is a^2, the *Species*, with its *coefficient*, $m + 1$.)
II. The *middle* term		The *Side* or *Root* (a) As $ba - ca$ (in which is a, the *Species*, with its *coefficient*, $b - c$.)
III. The *third* term.		1 or an *Unit*, As $e^2 + b^2 - nm$, which is *abſolutely* known *without* any dimenſion of a.

All theſe Equations which have three Species equally aſcending in the Scale, he reduceth to three Forms, (according as the ſigns +, — vary.)

$$\begin{array}{ll} 1.\ za - a^2 = æ & 2.\ a^2 - xa = æ \\ \quad za^2 - a^4 = æ^2 & \quad a^4 - xa^2 = æ^2 \\ \quad za^3 - a^6 = æ^3 & \quad a^6 - xa^3 = æ^3 \\ \quad \&c. & \quad \&c. \end{array}$$

$$\begin{array}{ll} \text{or, } ze - e^2 = æ & 3.\ e^2 + xe = æ \\ \quad ze^2 - e^4 = æ^2 & \quad e^4 + xe^2 = æ^2 \\ \quad ze^3 - e^6 = æ^3 & \quad e^6 + xe^3 = æ^3 \\ \quad \&c. & \quad \&c. \end{array}$$

In all Equations where the higheſt term is a^2 (or the Square of the Quantity ſought) he proves the third term to be æ (the Product of two Quantities, viz. a (the greater) and e (the leſſer), and that the Coefficient of the middle term in the firſt Form is z (the ſum of theſe two, a, e) but in the two other Forms that it is x (their difference) and this he doth as followeth,

$$\text{For ſince (by chap 14.)} \left\{ \begin{array}{l} 1.\ z - a = e \\ \text{or } z - e = a \\ 2.\ a - x = e \\ 3.\ e + x = a \end{array} \right\} \text{Therfore multiplying both by} \left\{ \begin{array}{l} a \\ e \\ a \\ e \end{array} \right\} \text{there will be} \left\{ \begin{array}{l} za - a^2 \\ ze - e^2 \\ a^2 - xa \\ e^2 + xe \end{array} \right\} = æ$$

By which it appears of what parts each Form doth conſiſt. And ſince in the firſt Form there be two Affirmative Roots, either of which will perform the Equation, he therefore calls ſuch as theſe ambiguous Equations, and though (as ſhall be ſhewn hereafter) the ſecond and third Forms have each of them a Negative as well as an Affirmative

Root, yet he gives an Account of the *Affirmative* Roots *only*, and therefore takes no notice of a *Fourth* Form.

4. $-a^2 - za = \text{æ}$ or $-e^2 - ze = \text{æ}$ } in which *both* the Roots are *Negative.*

Now having z and æ *given* (as in the *firſt* Form) and x and æ *given* (as in the *ſecond* and *third* Forms) he finds the *Affirmative* Roots of all the *three* Forms of Equations, wherein the *higheſt* term is the *Square* of the Quantity *ſought* : As followeth,

	1	$a = \frac{1}{2}z + \frac{1}{2}x$	by Chap. 14.
	2	$e = \frac{1}{2}z - \frac{1}{2}x$	
	3	$\text{æ} = \frac{1}{4}z^2 - \frac{1}{4}x^2$	
$3 + \frac{1}{4}x^2$	4	$\frac{1}{4}z^2 = \frac{1}{4}x^2 + \text{æ}$	
$4 - \text{æ}$	5	$\frac{1}{4}x^2 = \frac{1}{4}z^2 - \text{æ}$	
$\surd\ 5$	6	$\frac{1}{2}x = \surd : \frac{1}{4}z^2 - \text{æ} :$	
$\surd\ 4$	7	$\frac{1}{2}z = \surd : \frac{1}{4}x^2 + \text{æ} :$	
1, 6	8	$\frac{1}{2}z + \surd : \frac{1}{4}z^2 - \text{æ} : = \frac{1}{2}z + \frac{1}{2}x = a$	in the 1ſt. Form.
2, 6	9	$\frac{1}{2}z - \surd : \frac{1}{4}z^2 - \text{æ} : = \frac{1}{2}z - \frac{1}{2}x = e$	
1, 7	10	$\surd : \frac{1}{4}x^2 + \text{æ} : + \frac{1}{2}x = \frac{1}{2}z + \frac{1}{2}x = a$	in the *ſecond*
2, 7	12	$\surd : \frac{1}{4}x^2 + \text{æ} : - \frac{1}{2}x = \frac{1}{2}z - \frac{1}{2}x = e$	in the *third*

By theſe Rules may be diſcover'd the *middle* Species or Root of any other *Quadratic* Equation, As a^2, a^3, &c. But then after this, there muſt be *extracted* the reſpective Root of each, to find out a, the *Original* Root, or Quantity *ſought*.

For ſince (*chap.* 14.)

$$\left\{\begin{array}{l} 1.\ z - a^2 = e^2 \\ \quad z - a^3 = e^3 \\ \text{or } z - e^2 = a^2 \\ \quad z - e^3 = a^3 \\ 2.\ a^2 - x = e^2 \\ \quad a^3 - x = e^3 \\ 3.\ e^2 + x = a^2 \\ \quad e^3 + x = a^3 \end{array}\right\}$$

therefore multiplying both by

$$\left\{\begin{array}{l} a^2 \\ a^3 \\ e^2 \\ e^3 \\ a^2 \\ a^3 \\ e^2 \\ e^3 \end{array}\right\}$$

there will be

$$\left\{\begin{array}{l} za^2 - a^4 = a^2e^2 = a^2 \\ za^3 - a^6 = a^3e^3 = a^3 \\ ze^2 - e^4 = a^2e^2 = a^2 \\ ze^3 - e^6 = a^3e^3 = a^3 \\ a^4 - xa^2 = a^2e^2 = a^2 \\ a^6 - xa^3 = a^3e^3 = a^3 \\ e^4 + xe^2 = a^2e^2 = a^2 \\ e^6 + xe^3 = a^3e^3 = a^3 \end{array}\right.$$

This is the ſubſtance of what is in Mr. *Oughtred's* *Clavis* concerning *Quadratic* Equations.

Mr. *Hariot* hath a *different* Method of reſolving *theſe* Equations, and conſiders not only the *Poſſitive* or *Affirmative*, but alſo the *Privative* or *Negative* Roots; for ſuppoſing $aa = 9$, the *value* of a may be *either* $+ 3$, or $- 3$; ſince $+ 3 \times + 3$, or $- 3 \times - 3$ make 9, and *therefore* both are *true* Roots.

Now (in order to the diſcovering the *Riſe* and *Nature* of theſe *Compound* Equations) he conſiders the *ſingle* Equations of which they are *compoſed*, and puts each *ſingle* Equation *all over* on one ſide (which makes the whole *equal* to *o*) then *multiplying* them by each other, the *Product* will be *equal* to *o alſo*, As in the following *Inſtances*.

$$\begin{array}{r|l|l} \text{I.} & a=+b & a-b=0 \\ & a=+c & a-c=0 \\ \cline{3-3} & & aa-ba \\ & & \quad -ca+bc \\ \cline{3-3} & & aa-ba-ca+bc=0 \end{array}$$

$$\begin{array}{r|l|l} \text{II.} & a=+b & a-b=0 \\ & a=-c & a+c=0 \\ \cline{3-3} & & aa-ba \\ & & \quad +ca-bc \\ \cline{3-3} & & aa-ba+ca-bc=0 \end{array}$$

$$\begin{array}{r|l|l} \text{III.} & a=-b & a+b=0 \\ & a=-c & a+c=0 \\ \cline{3-3} & & aa+ba \\ & & \quad +ca+bc \\ \cline{3-3} & & aa+ba+ca+bc=0 \end{array}$$

Theſe he calls *Original* Equations, from which (by *tranſpoſing* the *laſt* term *bc*) he deduces others.

As from the $\left\{\begin{array}{l} 1^{st} \\ 2^{d} \\ 3^{d} \end{array}\right\}$ he deduces $\left\{\begin{array}{l} aa - ba - ca = - bc \\ aa - ba + ca = + bc \\ aa + ba + ca = - bc \end{array}\right\}$ which he calls *Canonical* Equations.

To the *three foregoing* Equations he adds *two more* as particular caſes; which he calls *Secondary Canonicals.*

$$\begin{array}{lll} \text{IV.}\ aa = + bc & aa - bc = 0 \\ \text{V.}\ aa = - bc & aa + bc = 0 \end{array}\Bigg\}\ \textit{that is,}$$

$$\left.\begin{array}{l} aa - 0a + 0a - bc = 0 \\ aa \mp 0a \mp 0a + bc = 0 \end{array}\right\}$$ whereby it appears that the *former* falls under the *ſecond* caſe, and the *latter* under the *firſt* or *third* caſe.

Here it is to be *obſerv'd* that a is not of the *ſame* value *quite* through each Equation, As that a which is ſuppoſed to be *equal to* b, is *not* the *ſame* with that which is *equal* to c, but of *another* value, *that is*, aa is not a^2, but the *Rectangle* or *Product* of two *different* a's. Whereas in every *Quadratic* Equation it is alwayes underſtood to be of the *ſame* value *throughout*. However *this Method* of *Compoſition* he makes uſe of for the better diſcovering how many *Roots* there are in a *Compound* Equation, and *what* they are, *Affirmative* or *Negative*, and *what* the *Coefficients* in each member are *made up of*.

And though theſe *Original* Equations are not *Quadratics*, yet they may be eaſily reduced to *ſuch*, and the Equations *ſtill* hold good: As,

I. In $aa - ba - ca + bc = 0$ putting $+b$ or $+c$ inſtead of a.	the *Equat.* will become	$bb - bb - cb + bc = 0$ or, $cc - cb - cc + bc = 0$
II. In $aa - ba + ca - bc = 0$ putting $+b$ or $-c$ inſtead of a.		$bb - bb + cb - bc = 0$ or, $cc + bc - cc - bc = 0$
III. In $aa + ba + ca + bc = 0$ putting $-b$ or $-c$ inſtead of a.		$bb - bb - cb + bc = 0$ or, $cc - bc - cc + bc = 0$

From whence it is evident that the Equations *ſtill* hold *good*. For in $aa - ba - ca + bc = 0$, as $aa - ba$ *deſtroys* it ſelf, *that is*, $aa - ba = 0$ (becauſe in *aa one* of the *a*'s is *equal* to *b*, and the *other* the *ſame* with *a* in *ba*) and $-ca + bc$ doth *alſo deſtroy* it ſelf, *a* being *equal* to *b*, ſo that the *whole* Equation becomes *equal* to *0*: So in $bb - bb - cb + bc = 0$, it is evident that $bb - bb$, and $-cb + bc$ *deſtroy* themſelves, and *conſequently* the *whole* is equal to *nothing*. Then by *tranſpoſing* *bc* in *both* the Equations there will be $aa - bb$

$-ca$

$-ca = -bc$ and $bb - bb - cb = -bc$, *that is*, $bb - :b + c: \times b = -bc$. And *thus* the *ſame* method of reaſoning will ſerve for *all* the reſt. So that each *Original* Equation may be ſuppoſed to have an a of the *ſame* value (whether of b or c) *quite* through, and thereby become a *Quadratic*.

Then (putting z for the *Sum*, x for the *difference* of b, c taken *abſolutely* without their ſigns) *theſe* Equations may be conſider'd as followeth,

$$\text{1. Caſe}\left\{\begin{array}{l} \qquad aa - ba - ca + bc = 0 \\ \textit{that is}, aa - :b + c: \times a + bc = 0 \\ \text{or}, \quad aa - za + bc = 0 \end{array}\right\} R^{ts}. +b, +c.$$

$$\text{2. Caſe}\left\{\begin{array}{l} \qquad aa - ba + ca - bc = 0 \\ \textit{that is}, aa \pm :c - b: \times a - bc = 0 \\ \text{or}, \quad aa \pm xa - bc = 0 \end{array}\right\} R^{ts}. +b, -c$$

$$\textit{that is},\left\{\begin{array}{l} -xa, \text{ when } b \text{ is } \textit{greater} \\ +xa, \text{ when } c \text{ is } \textit{greater} \\ 0 \text{ when they be } \textit{equal} \end{array}\right\}$$

$$\text{3. Caſe}\left\{\begin{array}{l} \qquad aa + ba + ca + bc = 0 \\ \textit{that is}, aa + :b + c: \times a + bc = 0 \\ \text{or}, \quad aa + za + bc = 0 \end{array}\right\} R^{ts}. -b, -c$$

From *theſe* Three Caſes it is evident that *ſuch Quadratic* Equations have *two* Roots, as here, b, c. And that the *known* Quantity (bc) in all of them is the *Product* of thoſe Roots *multiplyed* by each other.

And then (all being brought *over* to *one* ſide, and thereby *equal* to 0, and the *higheſt* power *Affirmative* and clear of *affection*, or without any *Coefficient*) if bc be *affirmative* ($+bc$) the *two* Roots have *like* ſigns (*that is*, both + or both —) as in the *firſt* and *third* Caſes wherein the *Coefficient* of the *middle* term is the *ſum* of *both* Roots with *contrary* ſigns, and therefore if the ſign of the *Coefficient* be —(as in the *firſt* Caſe) the Roots are both +, but if + (as in the *third* Caſe) both are —. *But* if *bc* be *negative* (as $-bc$) the Roots are *unlike* (the one +, the other —) as in the *ſecond* Caſe where the *middle* term ($-b+c$) is the *ſum* of *both* Roots with *contrary* ſigns, and conſequently *without* its ſign the *difference* of the *two* Roots ($c-b=x$) and *therefore* if the ſign of x (the *Coefficient*) be —, the *greater* of the *two* Roots (b) will be *affirmative* (becauſe in $c-b$ it is $-b$, and therefore in the Root $+b$.) But if the ſign of x be +, the *greater* Root (c) is *negative* (ſince in $c-b$) it is $+c$, and therefore in the Root $-c$.) And if the *middle* term be *wanting* or *equal* to 0, then is $b=c$, and therefore $-b+c=0$, and $-ba+ca=0$.

And ſince,

by ch. 14	1	$\frac{1}{2}z+\frac{1}{2}x=$ the *greater*	of the *two* Roots, *b*, *c*.
	2	$\frac{1}{2}z-\frac{1}{2}x=$ the *leſſer*	
1 × 2	3	$\frac{1}{4}zz-\frac{1}{4}xx=bc$	
$3+\frac{1}{4}x$	4	$\frac{1}{4}xx+bc=\frac{1}{4}zz$	
$4-bc$	5	$\frac{1}{4}zz-bc=\frac{1}{4}xx$	
$\sqrt{4}$	6	$\sqrt{:\frac{1}{4}xx+bc:}=\frac{1}{2}z$	
$\sqrt{5}$	7	$\sqrt{:\frac{1}{4}zz-bc:}=\frac{1}{2}x$	

From hence it follows that all (*poſſible*) *Quadratic* Equations may be reſolved *thus*,

I. $aa - za + bc = 0$ $\quad + \frac{1}{2}z \pm \sqrt{:}\,\frac{1}{4}zz - bc:$
II. $aa \pm xa - bc = 0$ $\quad \pm \frac{1}{2}x - \sqrt{:}\,\frac{1}{4}xx + bc:$
III. $aa + za + bc = 0$ $\quad - \frac{1}{2}z \pm \sqrt{:}\,\frac{1}{4}zz - bc:$

It is *obſervable* in the *ſecond* Caſe, that if x (the *difference*) be 0, xa will be *wanting*, and $\pm\frac{1}{2}x + \sqrt{:}\,\frac{1}{4}xx + bc: = \pm\frac{1}{2}0 + \sqrt{:}\,00 + bc: = \pm\sqrt{bc}$ and conſequently the Equation thence ariſing ($aa - bc$) will have *two* Roots $a = \pm\sqrt{bc}$.

And in the *firſt* and *third* caſes, if $\frac{1}{4}zz = bc$, then will $\sqrt{:}\,\frac{1}{4}zz - bc: = 0$; and therefore in the *firſt* caſe each Root $= + \frac{1}{2}z$; and in the *third* $= - \frac{1}{2}z$ but in theſe caſes if $\frac{1}{4}zz$ be *leſs* than bc, the Equation is *impoſſible*, and hath no *Real* but *Imaginary* Roots; For then $\frac{1}{4}zz - bc$ will be a *negative* Quantity; and therefore $\sqrt{:}\,zz - bc:$ will be the *Square* Root of a *negative* Quantity, which cannot be, ſince whether the Root be *Affirmative* or *Negative*, the Square will ſtill be *Affirmative*: As for inſtance, though 9 be a *Square* Number, whoſe Root may be + 3 or — 3, yet will not the *Square* of either be —9, but + 9.

And *thus*, this Equation (and ſuch like) $aa - bc$ or $aa + bc = 0$ (which belongs to the *firſt* or *third* Caſe wanting the *middle* term; $aa \pm 0a \pm bc = 0$) hath only *imaginary* Roots. $0 \pm \sqrt{:}\,0 - bc:$

Yet are not ſuch Equations as theſe altogether *uſeleſs*, ſince hereby may be diſcover'd not only the *impoſſibility* of the caſe propoſed, but alſo the *meaſure* of that impoſſibility, how *far* it is *impoſſible*, and what *alteration* would render the caſe *poſſible*.

Now though the foregoing *Method* of reſolving *Quadratic* Equations is what naturally follows from Mr. *Hariot*'s *Method* of compounding them, yet it is not the *ſame* with his which is a *peculiar* way of his own, and that is by adding the *Square* of half the *Coefficient* to each part of the Equation, thereby making the *unknown part* a *compleat* Square in *Species* equal to the *Abſolute given* Quantity. Thus in the ſeveral *Forms*, wherein half the *Coefficient* $(2b)$ is equal b.

$$\begin{array}{llll} \textit{To} & aa - 2ba = + cc & & \\ \textit{Add} & \quad + bb \qquad + bb & & a - b \\ & \overline{} & & a - b \\ & aa - 2ba + bb = + bb + cc & & \overline{} \\ & \overline{} & & aa - ab \\ \text{The Sq. Rt. is} & a - b = \pm \sqrt{:} + bb + cc & & \quad - ab + bb \\ \text{to which add} & \quad + b \qquad + b & & \overline{} \\ & \overline{} & & aa - 2ab + bb \\ \text{then will} & a = + b \pm \sqrt{:} + bb + cc : & & \end{array}$$

In *this* Equation the *greater* value of a (*that is*, $+ b + \sqrt{:} + bb + cc :$) is *Affirmative*; But the *leſſer* $(+ b - \sqrt{:} + bb + cc :)$ is *Negative*, becauſe the Root $\sqrt{:} + bb + cc :$ is *greater* than $b = \sqrt{bb}$

$$\begin{array}{lll} aa + 2ba = + cc & & \\ \quad + bb \qquad + bb & & a + b \\ \overline{} & & a + b \\ aa + 2ba + bb = + bb + cc & & \overline{} \\ \overline{} & & aa + ab \\ a + b = \pm \sqrt{:} + bb + cc : & & \quad + ab + bb \\ \quad - b \qquad - b & & \overline{} \\ \overline{} & & aa + 2ab + bb \\ a = - b \pm \sqrt{:} + bb + cc : & & \end{array}$$

Here the *leſſer* value of a (*that is,* $-b+\surd:bb+cc:$) is *Affirmative* but the *greater* ($-b-\surd;bb+cc:=-:+b+\surd:bb+cc:$) is *Negative*.

$$\begin{array}{l} aa-2ba=-cc\\ \quad\ +bb \qquad\ +bb\\ \hline aa-2ba+bb=bb-cc\\ \hline a-b=\pm\surd:bb-cc:\\ \quad+b\qquad+b\\ \hline a=+b\pm\surd:bb-cc: \end{array}$$

Both which values of a (if poſſible) *that is,* $+b+\surd:bb-cc:$ and $+b-\surd:bb-cc:$ are *Affirmative*; but may chance to be but *Imaginary*.

$$\begin{array}{l} aa+2ba=-cc\\ \quad\ +bb \qquad\ +bb\\ \hline aa+2ba+bb=+bb-cc\\ \hline a+b=\pm\surd:+bb-cc:\\ \quad-b\qquad-b\\ \hline a=-b\pm\surd:bb-cc: \end{array}$$

Both which values of a (if poſſible) are *Negative*; but *theſe* alſo may happen to be *Imaginary*. For in theſe two laſt Caſes) if cc be *greater* than bb, the Equation will be *impoſſible*, becauſe $bb-cc$ would then be a *negative* Quantity, and therefore $\surd:bb-cc:$ would be the *Square* Root of a *Negative* which cannot be, ſince whether the Root be *Affirmative* or *Negative*, the Square thereof will be Affirmative, and therefore theſe Roots (if poſſible) $\surd : + bb + cc :$ and $\surd : bb - cc :$ may be either *Affirmative* or *Negative* (*that is* $\pm \surd : bb + cc :$ and $\pm \surd : bb - cc :$) becauſe the *Square* of either will be *Affirmative* (*that is*, $+ bb + cc$ and $+ : bb - cc :$

It is to be *obſerved* that a is *Negative* in ſome of the aforegoing Equations, though their Forms appears like thoſe wherein it is *Affirmative*, as is manifeſt by the following Operations.

$$\begin{array}{rl}
 & -a - b \\
 & -a - b \\
\hline
 & + aa - b \times -a \\
 & \quad - b \times -a + bb \\
\hline
 & -a \times -a - 2b \times -a + bb \\
\textit{that is,} & aa - 2ba + bb
\end{array}$$

$$\begin{array}{rl}
 & -a + b \\
 & -a + b \\
\hline
 & -a \times -a + b \times -a \\
 & \quad + b \times -a + bb \\
\hline
 & -a \times -a + 2b \times -a + bb \\
\textit{that is,} & aa + 2ba + bb
\end{array}$$

There is *another Method* of reſolving *Quadratic* Equations, which is by caſting away the ſecond term, and this is done by *adding* half the *Coefficient* (b) to, or *ſubſtracting* it from the value of a (the Root *unknown*.) Thus,

putting

putting	1	$aa - 2ba = + cc$
	2	$a = e + b$
Q:2	3	$aa = ee + 2be + bb$
$2 \times 2b$	4	$2ba = + 2be + 2bb$
$3 - 4$	5	$ee - bb = aa - 2ba = + cc$
$5 + bb$	6	$ee = + bb + cc$
$\surd : 6$	7	$e = \pm \surd : bb + cc :$
$7 + b$	8	$a (= b + e) = + b \pm \surd : + bb + cc :$

putting	9	$aa + 2ba = + cc$
	10	$a = e - b$
Q: 11	11	$aa = ee - 2be + bb$
$10 \times 2b$	12	$2ba = + 2be - 2bb$
$11 + 12$	13	$ee - bb = aa + 2ba = + cc$
$13 + bb$	14	$ee = + bb + cc$
$\surd : 14$	15	$e = \pm \surd : bb + cc :$
$15 - b$	16	$a (= e - b) = - b \pm \surd : bb + cc :$

putting	17	$aa - 2ba = - cc$
	18	$a = e + b$
Q: 18	19	$aa = ee + 2be + bb$
$18 \times 2b$	20	$2ba = + 2be + 2bb$
$19 - 20$	21	$ee - bb = aa - 2ba = - cc$
$21 + bb$	22	$ee = + bb - cc$
$\surd : 22$	23	$e = \pm \surd : + bb - cc :$
$23 + b$	24	$a (= b + e) = + b \pm \surd : + bb - cc :$

putting	25	$aa + 2ba = - cc$
	26	$a = e - b$
Q: 26	27	$aa = ee - 2be + bb$
$26 \times 2b$	28	$2ba = + 2be - 2bb$
$27 + 28$	29	$ee - bb = aa + 2ba = - cc$
$29 + bb$	30	$ee = + bb - cc$
$\surd : 30$	31	$e = \pm \surd : + bb - cc :$
$31 - b$	32	$a (= e - b) = - b \pm \surd : + bb - cc :$

From what hath been ſaid, it is evident that the ſeveral *Forms* of *Affected Quadratic* Equations may be *reſolved*, as followeth.

Of theſe *Equations*. The *Roots* are

$$\left.\begin{array}{ll} 1.\ aa - 2ba = +cc. & a = +b \pm \sqrt{:+bb+cc:} \\ 2.\ aa + 2ba = +cc. & a = -b \pm \sqrt{:+bb+cc:} \end{array}\right\}\text{the greater}\left\{\begin{array}{l}\textit{Affirmative.} \\ \textit{Negative.}\end{array}\right.$$

$$\left.\begin{array}{ll} 3.\ aa - 2ba = -cc. & a = +b \pm \sqrt{:+bb-cc:} \\ 4.\ aa + 2ba = -cc. & a = -b \pm \sqrt{:+bb-cc:} \end{array}\right\}\text{both}\left\{\begin{array}{l}\textit{Affirmative.} \\ \textit{Negative.}\end{array}\right.$$

When the *Coefficient* ($2b$) though *not* a *Fraction* is an *odd* Number, b and bb will be *Fractions*, to prevent which, it will be convenient to put for the *Coefficient* $f = 2b$ and *double* the Roots, *Thus*,

The

The Equations	The *double* Roots
1. $aa - 2ba = + cc$	$2a = + f \pm \surd : + ff + 4cc :$
2. $aa + 2ba = + cc$	$2a = - f \pm \surd : + ff + 4cc :$
3. $aa - 2ba = - cc$	$2a = + f \pm \surd : + ff - 4cc :$
4. $aa + 2ba = - cc$	$2a = - f \pm \surd : + ff - 4cc :$

For ſince $f = 2b$ therefore $ff = 4b$ and

$$\overline{\surd : bb + cc} : \times 2 : \times \overline{\surd : bb + cc} : \times 2$$
$$= \overline{\surd : bb + cc : \times \surd : bb + cc} : \times 2 \times 2$$
$$= : bb + cc : \times 4 = 4bb + 4cc$$
$$\therefore \surd : 4bb + 4cc : (= \surd : ff + 4cc :) = \overline{\surd : bb + cc : \times 2}$$

And therefore

$$2a = f \pm \surd : ff + 4cc =$$
$$2b \pm \surd : \overline{bb + cc} : \times 2$$

Then taking half of this *double* Root for the value of a, *that is*, $a = \frac{+ f \pm \surd : + ff + 4cc :}{2}$ rather than $a = + \frac{1}{2} f \pm \surd : + \frac{1}{4} ff + cc :$ and ſo for the reſt.

And for the *like reaſon* when the *higheſt* power of a is *affected*; as in this Equation; $maa - lga = + ndd$, if m be not ſuch a quantity as will divide lg and ndd without a *Fraction*, ſo that $\frac{lg}{m} = f$ and $\frac{ndd}{m} = cc$ may be *Integers*: It may be as convenient (without dividing the whole by m for *clearing* aa from Affection) to expreſs ſuch Equations thus, The Equations.

$$maa - lga = + ndd$$
$$maa + lga = + ndd$$
$$maa - lga = - ndd$$
$$maa + lga = - ndd$$

The *Multiple* of the Roots.

$$2ma = + lg \pm \surd : llgg + 4mndd :$$
$$2ma = - lg \pm \surd : llgg + 4mndd :$$
$$2ma = + lg \pm \surd : llgg - 4mndd :$$
$$2ma = - lg \pm \surd : llgg - 4mndd :$$

And then divide the Multiple of the Root by $2m$ for a, that is, $a = \frac{+ lg \pm \surd : llgg + 4mndd}{2m}$ rather than $a = + \frac{lg}{2m} \pm \surd : \frac{llgg}{4m^2} + \frac{ndd}{m}$.

For $\frac{lg}{2m} \times 2m = lg$ and $\overline{\surd : \frac{llgg}{4m^2} + \frac{ndd}{m}} : \times 2m$ $= \surd : \overline{\frac{llgg}{4m^2} + \frac{ndd}{m} \times 4m^2} : = \surd : llgg + 4mndd :$

CHAP. XVI.

Of Arithmetical Progreſſions.

IN *Arithmetical Progreſſions* the terms do equally *increaſe* or *decreaſe*; as,

3, 5, 7, 9 &c. *increaſing*
16, 14, 12, 10 &c. *decreaſing*

In the former there is a continual *Increaſe*, in the latter a continual *decreaſe* by 2, which is called the *common difference* or *exceſs*.

And *univerſally* putting *A* for the *firſt* term, and *E* for the *common exceſs* or *difference* the terms will be

$A\ .\ A+E.\ A+2E.\ A+3E.$ &c. *increaſing.*
$A\ .\ A-E.\ A-2E.\ A-3E.$ &c. *decreaſing.*

But the moſt *ſimple* and *natural Progreſſion* is that which begins with 0, as,

$$\left.\begin{array}{lllll} 0. & E. & 2E, & 3E. & 4E \\ 0. & -E. & -2E, & -3E. & -4E \end{array}\right\}\text{\&c.}$$

When it begins with any *other* term (as *A* in the *former* Progreſſions) it is really a *Compound* of two Progreſſions; one of *Equals* (*A. A. A.* &c.) and the other of *Arithmetical Proportionals* (0. *E*. 2*E*. 3*E* &c.)

Now in order to the finding out how to *reſolve* Queſtions concerning *theſe* Progreſſions, let

a	ſignifie the	*leaſt* or *firſt* term.
u		*greateſt* or *laſt* term.
e		common *exceſs* or *difference*.
t		*Number* of terms.
d		*diſtance* of *one term* from *another*.
z		*Sum* of the *extremes* a and u.
x		*difference* of the *extr.* a and u,
s		*Sum* of *all* the terms.
A.B.C.D.&c.		Terms *beginning* with a
a. b. c.d.&c.		Terms beginning with u.
o. 1.2. 3.&c.		*Indexes* or *Numbers* ſhewing the *diſtance* of any term from a or u.

It is evident, from what hath been ſaid, that the terms of an *Arithmetical* Series *increaſing* ariſe by the continual *Addition*; but thoſe of a Series *decreaſing* by the continual *Subſtraction* of e (the *common difference.*) Thus,

0	*A*	$a \cdots = 3 = u - 9e$	k	9
1	*B*	$a + e = 5 = u - 8e$	i	8
2	*C*	$a + 2e = 7 = u - 7e$	h	7
3	*D*	$a + 3e = 9 = u - 6e$	g	6
4	*E*	$a + 4e = 11 = u - 5e$	f	5
5	*F*	$a + 5e = 13 = u - 4e$	e	4
6	*G*	$a + 6e = 15 = u - 3e$	d	3
7	*H*	$a + 7e = 17 = u - 2e$	c	2
8	*I*	$a + 8e = 19 = u - e.$	b	1
9	*K*	$a + 9e = 21 = u$	a	0

$s = 120 = \ldots s$

I.

Suppoſing d to be equal to the diſtance from a to u, then will

$$\begin{array}{r|l|l}
 & 1 & d = t - 1 \\
 & 2 & u = a + 9e = a + de \\
2 - a & 3 & u - a\ (= x) = de\ (= : t - 1 : \times e) = te - e \\
2 - e & 4 & u - e = a + 8e \\
4 - e & 5 & u - 2e = a + 7e \\
5 - e & 6 & u - 3e = a + 6e \\
\&c. & & \qquad \&c.
\end{array}$$

II.

$$\begin{array}{|llllllll|}
\hline
A + \mathrm{a} & = a & \ldots & + u & \ldots & = a + u & = z \\
B + \mathrm{b} & = a + e & & + u - e & & = a + u & = z \\
C + \mathrm{c} & = a + 2e & & + u - 2e & & = a + u & = z \\
D + \mathrm{d} & = a + 3e & & + u - 3e & & = a + u & = z \\
E + \mathrm{e} & = a + 4e & & + u - 4e & & = a + u & = z \\
F + \mathrm{f} & = a + 5e & & + u - 5e & & = a + u & = z \\
G + \mathrm{g} & = a + 6e & & + u - 6e & & - a + u & = z \\
H + \mathrm{h} & = a + 7e & & + u - 7e & & = a + u & = z \\
I + \mathrm{i} & = a + 8e & & + u - 8e & & = a + u & = z \\
K + \mathrm{k} & = a + 9e & & + u - 9e & & = a + u & = z \\
\hline
\end{array}$$

$$s + s = \ldots\ldots\ 2s\ \ldots\ldots = ta + tu = tz$$

That is, the *Sum* of the Extremes is equal to the *Sum* of any *two* termes *equidiſtant* from the Extremes (*that is*, when one term is as far diſtant from a as the other is from u) and conſequently if the Number of terms be *odd*, the *Double* of the *middle* term will be equal to the *Sum* of the Extremes becauſe it is *equidiſtant* from the Extremes, as if the Number of termes be 9, then will the *Middle* term (m) be $a+4e$ or $u-4e$, and therefore $a+4e+u-4e\ (=a+u)=m+m=2m$.

Alſo the *Sum* of the Extremes multiplied by the Number of terms is equal to twice the *Sum* of *all* the terms, $tz=2s$.

III.

If $A,\ M,\ V \div$ then will $2M = A+V$.
For, $A : M :: M : V$
that is, $A : A \pm e :: M : M \pm e$
therefore $A + M \pm e = M + A \pm e$
that is, $A + V = M + M = 2M$

Then,

Given	ſought			
		$2M = A+V$	1	
$M,\ A$		$V = 2M - A$	2	1 — A
$M,\ V$		$A = 2M - V$	3	1 — V
$V,\ A$		$M = \dfrac{A+V}{2}$	4	1 ÷ 2.

IV.

If $A : M :: N : V$ then will $A+V=M+N$.

For $A : M :: N : V$
that is, $A : A \pm e :: N : N \pm e$
Therefore $A + N \pm e = N + A \pm e$
that is, $A + V = N + M$.

Then,

			$A + V = M + N$	1	
Given	M,N,A	ſought	$V = M + N - A$	2	1 — A
	M,N,V		$A = M + N - V$	3	1 — V
	A,V,M		$N = A + V - M$	4	1 — M
	A,V,N		$M = A + V - N$	5	1 — N

V.

If out of an *Arithmetical Progreſſion* be taken any *Series* of *equidiſtant* terms, this *Series* will alſo be an *Arithmetical Progreſſion.*

As if $A . B . C . D . E . F . G . H . I . K$ &c. ÷

Then will

$$\left.\begin{array}{cccccccccc} A & . & C & . & E & . & G & . & I & \\ & B & . & D & . & F & . & H & . & K \\ A & . & & D & . & & G & . & & K \\ A & . & & & E & . & & & I & \end{array}\right\} \div$$

For ſince by the Nature of *Arithmetical Progreſſion*, in each *Series* every term *aforegoing* is ſo many e^s *more* or *leſs* than that next *following* as they are *diſtant* one from another, therefore being *equidiſtant* in each *Series* every term *aforegoing* will have a *like* Number of e^s *more* or *leſs* than that next *following*, and conſequently have *equal Differences* or *Exceſſes*.

VI.

If $a = e$ *that is*, if the *leaſt* term be equal to the *Common difference* then will

	1	$u = te$
1 ÷ t	2	$t)\,u = e$
1 ÷ e	3	$e)\,u = t$

VII.

VII.

If $d =$ the diſtance of any two terms (as B from F) then will $F - B = de$.

$$\begin{array}{l|l|l} \text{For} & 1 & F = B + de \\ 1 \div B & 2 & F - B = de \\ 2 \div d & 3 & \dfrac{F - B}{d} = e \\ 2 \div e & 4 & \dfrac{F - B}{e} = d \\ 1 - de & 5 & F - de = B \end{array}$$

VIII.

From the *firſt* and *ſecond* Sect. may be collected two *Lemma's*, by which may be reſolved *various* Queſtions concerning *Arithmetical Progreſſions*, *Viz.*

Lemma I. $u = a + e \times : t - 1 :$ or $a + te - e$.

Lemma II. $s = \frac{1}{2}\, t \times : a + u :$ or $\dfrac{t \times : a + u :}{2}$

Any *three* of which being given, the other *two* may be found out by the *foregoing* Lemma's. But for the reſolving of ſuch like Queſtions upon view the following *Synopſis* is very uſeful.

Put. a } = { First or Least Tearme
u } = { Last or Greatest Tearme
e } = { Comon Difference
t } = { Number of Tearmes
s } = { Sume of all ye Tearmes } as before

	data.	q.	Theorems.
1.	a,e,t	u	$=a+te-e$.
		s	$=\frac{2ta+tte-te}{2}$.
2.	a,e,u	t	$=\frac{u-a+e}{e}$.
		s	$=\frac{uu+ue-aa+ae}{2e}$.
3	a,e,s	t	$=\sqrt{\frac{2s}{e}+\frac{cc}{4}}-\frac{c}{2}$. Putting $\frac{2a-e}{e}=c$
		u	$=\sqrt{2es-ae+aa+\frac{ee}{4}}-\frac{e}{2}$
4	a,u,s	e	$=\frac{uu-aa}{2s-u-a}$.
		t	$=\frac{2s}{u+a}$.
5	a,t,s	e	$=\frac{2s-2ta}{tt-t}$.
		u	$=\frac{2s-ta}{t}$.

data		q.	Theorems.
6.	$a,t,u,$	$e.$	$=\frac{u-a}{t-1}.$
		$s.$	$=\frac{ta+tu}{2}.$
7.	$e,t,u,$	$a.$	$=u-te+e$
		$s.$	$=\frac{2tu-tte+te}{2}.$
8.	$e,t,s,$	$a.$	$=\frac{2s-tte+te}{2t}.$
		$u.$	$=\frac{2s+tte-te}{2t}.$
9.	$e,u,s,$	$a.$	$=\sqrt{uu+ue-2es+\frac{ee}{4}}+\frac{e}{2}.$
		$t.$	$=\frac{c}{2}-\sqrt{\frac{cc}{4}-\frac{2s}{e}}.$ Puting $\frac{2u+e}{e}=c$
10.	$t,u,s,$	$a.$	$=\frac{2s-tu}{t}.$
		$e.$	$=\frac{2tu-2s}{tt-t}.$

So that having any Three of yͤ aforeſaid Quantityes given the other Two are Likewiſe given.

CHAP. XVII.

Of Geometrical Progreſſions.

IN *Geometrical Progreſſions* the terms do either *increaſe* or *decreaſe* by *equal Proportions* or *Rations*, *that is*, according to one *common Multiplier* or *Diviſor* (the *Exponent* of the *common Ration.*) As,

2. 6. 12. 24 &c. *Increaſing.*
32. 16. 8. 4 &c. *Decreaſing.*

In the *former* 2 is the *common Multiplier*, in the *latter*, 2 is the *common Diviſor*, or (which is all one) $\frac{1}{2}$ is the *common Multiplier.*

And *univerſally*, putting A for the *firſt* term, and R for the *common Multiplier* or *Diviſor*, the terms will be

$$A.\ AR.\ AR^2.\ AR^3 \text{ \&c. } \textit{Increaſing.}$$
$$A.\ \frac{A}{R}.\ \frac{A}{R^2}.\ \frac{A}{R^3} \text{ \&c. } \textit{Decreaſing.}$$

In the *former* there is a *continual Multiplication* by R, in the *latter* a *continual Diviſion* by R (or a *continual Multiplication* by $\frac{1}{R}$.) But the moſt *natural* and *ſimple Progreſſion* is that which begins with 1. As,

$$1.\ R\ R^2 .\ R^3 .\ R^4 \text{ \&c.}$$

For when it begins with any *other* term (as *A* in the former *Progreſſions*) it only conſiſts of the terms of the *latter* each *multiplied* by *A*.

The *Exponents* or *Indexes* of the terms in this *latter* Progreſſion do make a rank of *Arithmetical Proportionals*: As,

$$\begin{array}{l} 0 \,.\, 1 \,.\, 2 \,.\, 3 \,.\, 4 \text{ \&c.} \\ 1 \,.\, R \,.\, R^2 .\, R^3 .\, R^3 \text{ \&c.} \end{array}$$

Wherein the *Addition* and *Subſtraction* of the *Exponents* anſwer to the *Multiplication* and *Diviſion* of the *Geometrical* terms, as $R \times R^2 \times R^3 = R^6$ whoſe Index 6 is equal to $1 + 2 + 3$ the Indexes of $R.\, R^2.\, R^3$, and upon *this Notion* is founded the whole Doctrine of *Logarithms*.

Now for the *clearer* diſcovery how Queſtions concerning *Geometrical Progreſſions* may be reſolved: Let,

a	ſignifie the	*Leaſt* or *firſt* term.
u		*Greateſt* or *laſt* term.
au		*Product* of the *Extremes*, $a \times u$.
$\frac{u}{a}$		*Proportion* of the *greateſt* to the *leaſt*.
z		*Sum* of *all* the *Products* of 2 terms *equidiſtant* from the Extremes.
t		*Number* of terms.
r		*Common Ration.*
d		*Diſtance* of one term from another.
s		*Sum* of *all* the *terms*.
A.B.C.&c.		Terms *beginning* with *a*.
a.b.c.&c.		Terms *beginning* with *u*.
0.1.2.&c.		*Indexes* or *Numbers* ſhewing the *diſtance* of any term from *a* or *u*.

Then,

Note, By comparing *chap.* next aforegoing with this, it is eaſie to diſcern, that *Multiplication* and *Diviſion*, *&c.* in *this* anſwers to *Addition*, *Subſtraction*, *&c.* in *that*.

Thus,

0	*A*	$a. =$	$3 = r^9)\,u$	k	9
1	*B*	$ar =$	$6 = r^8)\,u$	i	8
2	*C*	$ar^2 =$	$12 = r^7)\,u$	h	7
3	*D*	$ar^3 =$	$24 = r^6)\,u$	g	6
4	*E*	$ar^4 =$	$48 = r^5)\,u$	f	5
5	*F*	$ar^5 =$	$96 = r^4)\,u$	e	4
6	*G*	$ar^6 =$	$192 = r^3)\,u$	d	3
7	*H*	$ar^7 =$	$384 = r^2)\,u$	c	2
8	*I*	$ar^8 =$	$768 = r\ )\,u$	b	1
9	*K*	$ar^9 =$	$1536 = ..\ u$	a	0

$$s = 3069 = s$$

I.

Suppoſing *d* to be eqnal to the *diſtance* from *a* to *u*, then will

	1	$d = t - 1$
	2	$u = ar^9 = ar^d$
$2 \div a$	3	$a)\,u = \left(\frac{u}{a}\right) = r^d$
$2 \div r$	4	$r)\,u = \frac{ar^9}{r} = ar^8$
$4 \div r$	5	$r^2)\,u = \frac{ar^9}{r^2} = ar^7$
$5 \div r$	6	$r^3)\,u = \frac{ar^9}{r^3} = ar^6$

&c.

II.

II.

$$
\begin{array}{|lllllllll|}
\hline
A \times a = a & \times & & u = & \ldots & & & \ldots & au \\
B \times b = ar & \times & r\,) & u = & a & \times & u & = & au \\
C \times c = ar^2 & \times & r^2) & u = & a & \times & u & = & au \\
D \times d = ar^3 & \times & r^3) & u = & a & \times & u & = & au \\
E \times e = ar^4 & \times & r^4) & u = & a & \times & u & = & au \\
F \times f = ar^5 & \times & r^5) & u = & a & \times & u & = & aa \\
G \times g = ar^6 & \times & r^7) & u = & a & \times & u & = & au \\
H \times h = ar^7 & \times & r^7) & u = & a & \times & u & = & au \\
I \times i = ar^8 & \times & r^8) & u = & a & \times & u & = & au \\
K \times k = ar^9 & \times & r^9) & u = & a & \times & u & = & au \\
\hline
\end{array}
$$

$$z \ldots\ldots\ldots = ta \times tu : = tau$$

III.

If $A \,.\, M \,.\, V \div$ then will $M^2 = AV$.

For ſince $A : M :: M : V$.

$$\begin{array}{rl} \textit{that is}, & A : \left.\begin{array}{l} Ar \\ \frac{A}{r} \end{array}\right\} :: M : \left\{\begin{array}{l} Mr \\ \frac{M}{r}. \end{array}\right. \\ \text{or} & \end{array}$$

$$\text{therefore } AMr = MAr$$

$$\text{or } A\frac{M}{r} = M\frac{A}{r}$$

$$\textit{that is } AV = MM = M^2$$

Then,

Given	Sought		
	$M^2 = AV$	1	
M, A	$V = \frac{M^2}{A}$	2	$1 \div A$
M, V	$A = \frac{M^2}{V}$	3	$1 \div V$
A, V	$M = \sqrt{:}\ AV$	4	$\sqrt{}\ 1$

IV.

If $A : M :: N : V$ then will $AV = MN$.

For $A : M :: N : V$.

that is, $A : Ar :: N : Nr$.

or $A : \frac{A}{r} :: N : \frac{N}{r}$

Therefore $ANr = NAr$

or $A\frac{N}{r} = N\frac{A}{r}$

that is, $AV = NM$.

Then;

Given	Sought		
	$AV = MN$	1	
$M, N, A.$	$V = \frac{MN}{A}$	2	$1 \div A$
$M, N, V.$	$A = \frac{MN}{V}$	3	$1 \div V$
$A, V, M.$	$N = \frac{AV}{M}$	4	$1 \div M$
$A, V, N.$	$M = \frac{AV}{N}$	5	$1 \div N$

V.

V.

If out of a *Geometrical Progreſſion* be taken any *Series* of *Equidiſtant* terms, *this* Series will alſo be ∺.

As if $A . B . C . D . E . F . G . H . I . K$ &c. ∺

$$\begin{array}{l} \text{Then} \\ \text{will} \end{array}\left\{\begin{array}{cccccccccc} A & . & C & . & E & . & G & . & I & \\ & B & . & D & . & F & . & H & . & K \\ A & & . & D & . & & G & & . & K \\ A & & . & & E & & . & & I & \end{array}\right\} ∺$$

For in *each* reſpective Series the terms being *equidiſtant* there will be the *ſame* r^d, *that is*, the *ſame* ration according to the *diſtance* of the terms in each Series. As for inſtance in the *laſt* Series,

$$A ; E = Ar^4 \text{ or } r^4) A :: E : I = Er^4 \text{ or } r^4) E.$$

VI.

If	1	$a = r$
then will	2	$u = r^t$
$\sqrt{}^t\ 2$	3	$\sqrt{}^t u = r$
$r \div) 2$	4	$r \div) u = t$

VII.

If $d =$ the diſtance of two terms from one another (as B from F) then will $F) B = r^d$; For,

	1	$F = Br^d$
$1 \div B$	2	$B) F = r^d$
$r \div) 2$	3	$r \div) \frac{F}{B} = d$
$\sqrt{}^d : 2$	4	$\sqrt{}^d : B) F : = r$
$1 \div r^d$	5	$r^d) F = B$

In any Geometricall Progreſsion Increasing, as, $a . ar . arr . arrr$ &c or Decreasing as $a . \frac{a}{r} . \frac{a}{rr} . \frac{a}{rrr}$. &c.

Puting
r — Ratio.
a — First or Laſt or Leaſt Term in ye Former or Latter Case
l — Laſt or Firſt or Greateſt Term in ye Former or Latter Case
s — Sume of all ye Termes. Then

data	quiſit	Theorem:	Demonſt:
r, l, a,	s.	$= \frac{rl - a}{r - 1}$.	For by Theo: 6th ye 9th Chap: 8th.
s, a, l,	r.	$= \frac{s - a}{s - l}$.	$1 . r :: s - l . s - a$
s, r, l,	a.	$= s + rl - rs$.	$\therefore s - a = sr - lr$
r, s, a,	l.	$= \frac{rs - s + a}{r}$.	q.e.d;

In any Decreaſing Geom: Progreſsion, Infinitely continu'd, untill ye Laſt or Leaſt Terme be $= 0$. Then

data	quiſit	Theorem.	Demonſt:
l, r,	s.	$= \frac{lr}{r - 1}$.	By Theo: 9th Cha ye 8th.
s, r,	l.	$= \frac{sr - s}{r}$.	$1 . r :: s - l ; s - 0$
s, l,	r.	$= \frac{s}{s - l}$.	$\therefore s = rs - rl$
			q. e. d.

So that ye Sume of such Progreſsion Infinitely Continued, is but a Finite Quantity.

CHAP. XVIII.

Of Infinite Geometrical Progreſſions.

AN *Infinite Geometrical* Progreſſion is that which is *infinitely* continued; or whoſe Number of terms are *infinite*.

1. If the *Proportion* of the terms one to another be that of *Equality* (*that is*, as 1 to 1, or *Equals*) or that of *Majority* (*that is*, if the *firſt* be to the *ſecond*; the *ſecond* to the *third*, &c. as 1 to *more* than 1, or if the terms *continually increaſe*) it is evident that the *Sum* of the Terms in *both* theſe caſes muſt be *infinitely great*. For an *infinite* Number of *equal* Quantities muſt needs be *infinite*, and much more that of Quantities *more* than equal.

2. If the *Proportion* be that of *Minority* (*that is*, if the *firſt* be to the *ſecond*, the *ſecond* to the *third*, &c. As 1 to *leſs* than one, or if the terms do *continually decreaſe*), then will the *Sum* be a *finite* Number. For ſince the terms do *continually decreaſe*) than will the *Sum* be a *finite* Number. For ſince the terms do *continually decreaſe* the *laſt* or *leaſt* term will be *infinitely ſmall*, or in *effect nothing*, and ſince (*Chap.* 8. *Theor.* 9.) in *finite* Progreſſions *decreaſing* $\frac{ur-a}{r-1}=s$ therefore in thoſe that are *infinite* $\frac{ur-a}{r-1}=s$ will be $\frac{ur-0}{r-1}=s$ *that is*, $\frac{ur}{r-1}=s$. Thus, if $a.b.c.d$ &c. ∺

then will (putting $r = \frac{a}{b}$) $\frac{ar}{r-1} = s = a + b + c + d$ &c. Or by the ſame reaſon ſince, (*Chap.* 8. *Theor.* 9.) $\frac{A - a : \times A}{A - B} = s - a$, therefore in *infinite* Progreſſions it will be $\frac{A - B : \times A}{A - B} = s - 0$ *that is*, $\frac{A^2}{A - B} = s$, and thus (*Chap.* 8. *Theor.* 9.) $\frac{u - a}{r - a} = s - u$ will *infinite* Progreſſions be $\frac{u - 0}{r - 1} = s - u$ *that is* $\frac{u}{r - 1} = s - u$. As $u + \frac{u}{r} + \frac{u}{r^2} + \frac{u}{r^3}$, &c. $- u = \frac{u}{r} + \frac{u}{r^2} + \frac{u}{r^3}$ &c. $\frac{u}{r - 1}$. So that the *Sum* of an *infinite* Number of terms *continually decreaſing* in a *Geometrical* Proportion will be a *finite* Number. *W. W. D.*

3. If a Number *given* (u) be continually *divided*

$$\text{by}\left\{\begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ \text{\&c.} \end{array}\right\} \text{the ſum of all theſe terms will be} = \left\{\begin{array}{ccc} \frac{u}{2-1} = \frac{u}{1} & \text{or} = & \\ \frac{u}{3-1} = \frac{u}{2} & & \frac{1}{2} \\ \frac{u}{4-1} = \frac{u}{3} & & \frac{1}{3} \\ \frac{u}{5-1} = \frac{u}{4} & & \frac{1}{4} \\ \text{\&c.} \quad \text{\&c.} & & \text{\&c.} \end{array}\right\} \text{of the Numb. given.}$$

If of the Number *given* (u) there be taken $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c. *that is*, if it be continually *Multiplyed*

by
$$\left\{\begin{array}{c} \frac{2}{3} \\ \frac{3}{4} \\ \frac{4}{5} \\ \frac{5}{6} \\ \frac{3}{7} \\ \&c. \end{array}\right\}$$

or (which is all one) continually *divided* by

$$\left\{\begin{array}{c} \frac{3}{2} \\ \frac{4}{3} \\ \frac{5}{4} \\ \frac{6}{5} \\ \frac{7}{3} \\ \&c. \end{array}\right\}$$

then will

$$\left\{\begin{array}{l} \frac{u}{\frac{3}{2}-1} = \frac{1}{2})\,u = 2u \\ \frac{u}{\frac{4}{2}-1} = \frac{1}{3})\,u = 3u \\ \frac{u}{\frac{5}{4}-1} = \frac{1}{4})\,u = 4u \\ \frac{u}{\frac{6}{5}-1} = \frac{1}{5})\,u = 5u \\ \frac{u}{\frac{7}{3}-1} = \frac{4}{3})\,u = \frac{3}{4}u \\ \&c. \end{array}\right\}$$

The *Sum* of all the terms.

The *truth* of *this* Propoſition is evident from that *next* aforegoing. So that if the Proportion be that of *Minority*, the *Sum* of the *whole* Progreſſion *infinitely* continued will alwayes be a *finite* Quantity: But *greater* than the Quantity *given* (u), if the part taken be *more* than $\frac{1}{2}$ thereof;

leſſer if *leſſer*, and *equal* if *equal* $\frac{1}{2}$. And alwayes as $r-1$ is to 1, ſo is u, to the *Sum*; *that is*, as the *ration* (or number by which that *given* is continually divided) *leſs* by an *Unit*, is to an *Unit*, ſo is the Number *given* to the *Sum* of the *whole* Progreſſion *infinitely* continued.

4. If *two* ſuch *Series* or *Progreſſions* be *compounded*, *that is*, (for inſtance) if there be taken $\frac{1}{b}$ of one Quantity (A) multiplied into $\frac{1}{c}$ of another (B) and again $\frac{1}{b}$ of that part into $\frac{1}{c}$ of *this* part, or $\frac{1}{b}A \times \frac{1}{B}\left(=\frac{1}{b}\times\frac{1}{c}AB=\frac{1}{bc}AB\right)$ and $\frac{1}{b}\times\frac{1}{b}A\times\frac{1}{c}\times\frac{1}{c}B\left(=\frac{1}{bb}\times\frac{1}{cc}AB=\frac{1}{bbcc}AB\right)$ and ſo on *infinitely*; or (which is the ſame thing) if the terms of the one Series $\left(\frac{1}{b}+\frac{1}{bb}A+\frac{1}{bbb}A\text{\&c.}\right)$ be reſpectively multiplied by thoſe of the other $\left(\frac{1}{c}+\frac{1}{cc}B+\frac{1}{ccc}B\right.$ &c. The *Sum* of this *compound* Progreſſion $\frac{1}{b}\times\frac{1}{c}AB+\frac{1}{bb}+\frac{1}{cc}AB+\frac{1}{bbb}\times\frac{1}{ccc}AB$, &c. *that is*, $\frac{AB}{bc}+\frac{AB}{bbcc}+\frac{Ab}{bbbccc}$ &c.

will *equal* $\frac{AB}{bc-1}$. For here the *Compound* of two Quantities (as of AB) is continually *divided* by the *Ration* bc, or continually *multiplied* by $\frac{1}{bc}$, which is a Proportion of Minority.

5. If a Series $\left(\frac{1}{b}A+\frac{1}{bb}A+\frac{1}{bbb}A\ \&c.\right)$ whoſe Proportion $\left(\frac{1}{b}\right)$ is that of Minority to be compounded (as in *Prop.* 4.) with another Series $\left(\frac{c}{1}B+\frac{cc}{1}B+\frac{ccc}{1}B\ \&c.\right.$ *that is*, $Bc+Bcc+Bccc$ &c.) whoſe Proportion $\left(\frac{c}{1}\right)$ is that of Majority, and if c be leſs than b, in this caſe (notwithſtanding the Sum of the latter Progreſſion be infinite) the Sum of the Compound Progreſſion, $\left(\frac{c}{b}AB+\frac{cc}{bb}AB+\frac{ccc}{bbb}AB\ \&c.\right)$ will be finite. For herein the Proportion $\frac{c}{b}$ or $\left.\frac{1}{c)b}\right)$ is that of Minority.

6. But if c be equal to or greater than b, in this caſe (the Proportion $\frac{c}{b}$ being that of Equa-

lity

lity or Majority) the Sum will be infinite (by *Prop.* 1.)

7. If of a Quantity given be taken $\frac{1}{b}$ of the whole, and then $\frac{1}{b}$ of the Remainder; and ſo on infinitely: The parts thus taken will equal the whole. As if $\frac{1}{b} = \frac{1}{3}$, this being taken away, the firſt remainder will be $\frac{2}{3}$; And $\frac{1}{3}$ of this being taken away, the ſecond Remainder will be $\frac{2}{3}$ of the firſt, *that is*, $\frac{2}{3} \times \frac{2}{3}$: And $\frac{1}{3}$ of this being taken away, the third Remainder will be $\frac{2}{3}$ of the ſecond, that is, $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$. Or univerſally thus, putting $b - 1 = e$, then will the Remainder be $\frac{e}{b}$. For the whole being $\frac{b}{b} = 1$ the Remainder is $\frac{b}{b} - \frac{1}{b} = \frac{b-1}{b} = \frac{e}{b}$. So that taking $\frac{1}{b}$ of the whole, the Remainder is $\frac{e}{b}$, then $\frac{1}{b}$ of this is $\frac{1}{b} \times \frac{e}{b} = \frac{e}{bb}$, and the Remainder $\frac{e}{b} \times \frac{e}{b} = \frac{ee}{bb}$, and then $\frac{1}{b}$ of this

&c.

&c. Now as e is to 1, ſo is $\frac{e}{b}+\frac{ee}{bb}+\frac{eee}{bbb}$ &c. to $\frac{1}{b}+\frac{e}{bb}+\frac{ee}{bbb}$, &c. (each term in this Series multiplyed by $\frac{e}{1}$ being reſpectively equal to each in that) (by conſtruction $\frac{e}{b}$ being leſs than 1.) But $\frac{e}{b}+\frac{ee}{bb}+\frac{eee}{bbb}$, &c. is $=\frac{1}{e)b,-1}=\frac{e}{b-e}=\frac{e}{1}=e$ (for $\frac{1\times e}{:e)b,-1:\times e}=\frac{e}{b-e}$ and $b=e+1$) therefore $e:1::e$ (the Sum of $\frac{e}{b}+\frac{ee}{bb}+\frac{eee}{bbb}$, &c.) : $\frac{1\times e}{e}=\frac{e}{1}=1$ (the Sum of $\frac{1}{b}+\frac{e}{bb}+\frac{ee}{bbb}$, &c.) *W. W. D.*

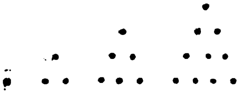
8. In like manner, if *Two Progreſſions* be ſo *Compounded* that of the two given Quantities *A* and *B*, if of the Quantity *A* be taken $\frac{1}{b}$ of the whole, and then $\frac{1}{b}$ of the Remainder, *&c.* and of the other Quantity *B* be taken $\frac{1}{c}$ of the whole, and then $\frac{1}{c}$ of the Remainder, *ad Infinitum*, The whole

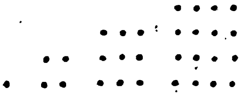
Series will be $= \frac{1}{bc - c + 1} \cdot AB$. For the firſt *Series* will be by the foregoing $\frac{1}{b} + \frac{1}{bb} + \frac{1}{bbb} +$&c. and the *Latter* (putting $c - 1 = d$) $= \frac{1}{c} + \frac{d}{cc} + \frac{dd}{ccc} +$ &c. and when reſpectively Multiplyed into each other will form this *Compound* one $\frac{1}{bc} + \frac{d}{bbcc} + \frac{dd}{bbbccc} +$ &c. $= \frac{1}{bc - c + 1}$, which by the 5° of this, is a Proportion of *Minority*, and by the third, to the ſum of the whole *Progreſſion*, as d, to 1. Therefore as $d.\ 1 :: \frac{d}{bc} + \frac{dd}{bbcc} + \frac{ddd}{bbbccc}$ + &c. $\left(= \frac{1}{bc - 1} = \frac{d}{bc - d} = \frac{d}{bc - c + 1}\right)$ to $\frac{1}{bc} + \frac{d}{bbcc} + \frac{dd}{bbbccc} +$ &c. $\left(= \frac{1}{bc - c + 1}\right.$ $A \times B$. *q. e. d.*

CHAP. XIX.

Of Poligonal Numbers.

POligonal or *Multangular* Numbers are the Sums or Aggregates of a Rank of Numbers in *Arithmetical Progreſſion* from Unity, and are ſo called, becauſe they repreſent the Number of *Points*, that are required to fill ſuch regular *Poligons* at equal diſtances on *Lines* drawn parallel to the *ſides* of the Figure: As,


&c. are Triangulars.


&c. are Quadrangulars, &c.

and are formed thus,

No. in Arith. Progreſſion.	*Whoſe com. Differ. is*	*Sums added from* 1.	*Poligon.*
1,2,3,4 ,&c.	1	1,3,6 ,10, &c.	Triangulars
1,3,5,7 ,&c.	2	1,4,9 ,16, &c.	Quadrang.
1,4,7,10,&c.	3	1,5,12,22, &c.	Pentangul.
1,5,9,13,&c.	4	1,6,15,28, &c	Hexangul.
	&c.		

Hence by Inſpection, theſe two Obſervations are evident;

1. That the *common Difference* will be always *equal* to the number of *Angles* leſs two.
2. That the *ſide* of the *Poligon* is *equal* to the number of *Terms* which compoſe the Figure.

Therefore putting $d =$ Common Difference.
$n =$ Number of Terms.
$l =$ Laſt or greateſt Term.
$z =$ Sum of all the Terms.
$p =$ *Poligon.*
$a =$ Number of Angles.

We have $p = \frac{\overline{nn - n} \times d}{2} + n$ Univerſally.

DEMONSTRATION.

Since every *Poligon* is the *aggregate* of Numbers in *Arithmetical Progreſſion*, whoſe firſt *Term* is Unity, by Vertue of the ſecond Lemma, *Theor.* 8. *Chap.* 16. $p = \frac{\overline{1 + l} \times n}{2}$, and by the firſt Lemma $l = \overline{1 + d} \times \overline{n - 1} = 1 + nd - d$, which ſubſtitute in the place of l in the former *Theorem* of the value of p, and we have $\frac{\overline{1 + 1 + nd - d} \times n}{2}$,

$\therefore p = \frac{\overline{nn - n} \times d}{2} + n$ Univerſally. *q. e. d.*

And becauſe d is given (by *Obſ.* 1.) in caſe its Value be *ſubſtituted* in this General *Theorem*, we may deduce particular ones for each *Poligon*. As in a

$$\left.\begin{array}{l}\text{Triang.}\\[2ex]\text{Quadr.}\\[2ex]\text{Pentan.}\\[2ex]\text{Hexan.}\end{array}\right\} d = \left\{\begin{array}{c}1\\[2ex]2\\[2ex]3\\[2ex]4\end{array}\right\} \text{and } p = \frac{2n + nnd - nd}{2} \therefore p = \left\{\begin{array}{c}\frac{nn+n}{2}\\[2ex] nn\\[2ex]\frac{3nn-n}{2}\\[2ex] 2nn - n\end{array}\right\} \text{q. e. d.}$$

&c.

Corol. The *common Difference* in a Rank or Series of *Poligons* is always $= \frac{nn - n}{2}$, which have the ſame ſide, or are Compoſed of the ſame number of *Terms*. Therefore,

By vertue of the ſaid Lemma's (*Theor.* 8. *Chap.* 16.) the *ſum* of any ſuch Series or Rank of *Poligonal Numbers* will always be $= \frac{\overline{n+p} \times \overline{a-1}}{2}$ $= \frac{an + ap - n - p}{2}$ Univerſally putting $p =$ Greateſt or Laſt Term, and $a =$ Angles of it, Then $a - 1 =$ Number of Terms, and $n =$ Firſt or Leaſt Term. And if we ſubſtitute a in Numbers in the Equation, we have *particular Theorems* for any Series: As in a

Triang.

$$\left.\begin{array}{l}\text{Triang.}\\ \text{Quadr.}\\ \text{Penta.}\\ \text{Hexan.}\end{array}\right\} a = \left\{\begin{array}{c}3\\ 4\\ 5\\ 6\end{array}\right\} \text{\&c. \& ſum} = \frac{an+ap-n-p}{2} = \left\{\begin{array}{c} n+p \\ \frac{2n+2p}{2} \\ 2n+2p \\ \frac{5n+5p}{2}\end{array}\right\} q.\,e.\,d.$$

But becauſe n may not always be given, ſince we find $p = \frac{2n+nnd-nd}{2}$, if we *compleat* the *Square* (as is directed *Page* 85. *Chap.* 5.) we ſhall find n (= ſide of any *Poligon*) $= \frac{d-2+\sqrt{dd-4d+4+8dp}}{2d}$ Univerſally. And becauſe d is given by (*Obſ.* 1.) if we ſubſtitute its *Value*, we have particular *Theorems* in this Caſe likewiſe. *Viz.* In a

$$\left.\begin{array}{l}\text{Triang.}\\ \text{Quadr.}\\ \text{Penta.}\\ \text{Hexan}\end{array}\right\} d = \left\{\begin{array}{c}1\\ 2\\ 3\\ 4\end{array}\right\} \text{\&c. and } n = \frac{d-2+\sqrt{ad-4d+4+8dp}}{2d} = \left\{\begin{array}{c} \frac{-1+\sqrt{1+8p}}{2} \\ \frac{0+\sqrt{0+10p}}{4} \\ \frac{1+\sqrt{1+24p}}{6} \\ \frac{2+\sqrt{4+32p}}{8}\end{array}\right\} q.\,e.\,d.$$

Thus having ſhewed how from the *ſide* given to find the *Poligon*, or from the *Poligon* to find the *ſide*, as alſo the *ſum* of any Series or Rank of *Regular Poligons* from a *Lateral*. I ſhall now give one *Univerſal Theorem* for finding any *Trigonal*, *Piramidal*, or other Number under-mentioned: As,

1.	Unity	1,	1,	1,	1,	1,	1
2.	Roots or Sides	1,	2,	3,	4,	5,	6
3.	Trigonals	1,	3,	6,	10,	15,	21
4.	Piramidals	1,	4,	10,	20,	35,	56
5.	Trig. Trigon.	1,	5,	15,	35,	70,	126
6.	Trig. Piramid.	1,	6,	21,	56,	126,	252
7.	Pira. Piramid.	1,	7,	28,	84,	210,	462
8.	Tri. Tri. Piram.	1,	8,	36,	120,	330,	792

&c.

Where every Number is the ſum of the preceeding Rank ſo far.

To find *any of which*, if we put the *Root* or *Side* of the given Rank $= r$, and the *Number* or *Place* required $= n$, and $r + n = m$. Then,

$$1 \times \frac{m-2}{1} \times \frac{m-3}{2} \times \frac{m-4}{3} \times \frac{m-5}{4} \times \frac{m-6}{5} \text{ \&c.}$$

will produce the ſame. *Examp.* To find the 5th *Piramidal*, becauſe $r = 4$ and $n = 5$ $\therefore m = 9$.

$\therefore 1 \times \frac{9-2}{1} \times \frac{9-3}{2} \times \frac{9-4}{3} = 35$, the *Piramidal* ſought; taking Notice, ſo many *Members* of this *Theorem* (continued if need be) muſt be uſed, as is the *Root* or *Side* of the given Rank, as in this laſt Caſe it was 4; and therefore 4 *Members* produced the Number required.

Wherefore ſince the *Roots* or *Sides* of ſuch *Ranks of Numbers* is always given, conſequently the *Number of Members* to be uſed in the *Theorem* for finding any Number in ſuch *Rank*, is likewiſe given, and thereby we have *particular Theorems* for every *Rank* by multiplying ſuch *Members*: As,

$$\left.\begin{array}{l}\text{Unity}\\[6pt] \text{Sides}\\[6pt] \text{Trigon.}\\[6pt] \text{Piramid.}\\[6pt] \text{Tri. Tri.}\\[6pt] \text{Tri. Pir.}\end{array}\right\} = \left\{\begin{array}{l} 1\\ \dfrac{m-2}{1}\\ \dfrac{mm-5m+6}{2}\\ \dfrac{m^3-9m^2+26m-24}{6}\\ \dfrac{m^4-14m^3+71m^2-154m+120}{24}\\ \dfrac{m^5-20m^4+155m^3-580m^2+1044m-720}{120}\end{array}\right.$$

&c.

Corol. Since every *Number* in any Rank is the *ſum* of all the *Numbers* in the preceding *Rank* ſo far, it is evident we have the *ſum* of any *requir'd Rank*, by finding the *Number* in the next *Rank* ſo far.

The *Demonſtration* of this *Univerſal Theorem*, would be too large for this place.

CHAP. XX.

Of working by the Exponants and Coefficients of Quantities.

1. IT is evident from *Page* 101, *Chap.* 17. That *Multiplying* or *Dividing* of *Powers*, may be done by *Adding* or *Subtracting* their *Exponants* reſpectively : As,

$$N^2 \times N^5 = N^7 \text{ or } N^{-2} \times N^5 = N^3$$
$$\text{or } N^2 \div \text{by } N^5 = N^{-3} \text{ or } N^{-2} \div \text{by } N^5 = N^{-3}$$

As likewiſe from the Rules for *Addition* and *Subtraction* of *Species* it is evident that,

$$4N - 3N = N \text{ or } 4N + 3N = 7N$$
$$\text{or } 7N - 4N = 3N \text{ or } 7N + 4N = 11N$$

Therefore put N = any aſſumed Number,
$\mathbf{n}$ = its Exponant,
n = its Coefficient.

And thereby we have a more *General* way of *Expreſſing* any *Increaſing* or *Decreaſing Rank* or *Series* of *Powers*, or any *Theorem* raiſed from the *Aggregates*, *Subductions*, *Rectangles* or *Quotes*, of ſuch *Quantities* or *Powers*, and a ſhorter way of performing the ſame, then can poſſibly be done by the *Common* way of *Notation*.

For by this Method

$$N^{n} \quad N^{n+1} \quad N^{n+2} \quad N^{n+3}$$

may repreſent

$$\left.\begin{array}{lllll} & N^{1} & N^{2} & N^{3} & N^{4} \\ \text{or} & N^{3} & N^{4} & N^{5} & N^{6} \\ \text{or} & N^{6} & N^{7} & N^{8} & N^{9} \end{array}\right\} \text{putting } n = \left\{\begin{array}{c} 1 \\ 3 \\ 6 \end{array}\right.$$

&c.

Or $N^{-n} \quad N^{-n+1} \quad N^{-n+2} \quad N^{-n+3}$.

may repreſent

$$\left.\begin{array}{lllll} & N^{-2} & N^{-1} & N^{0} & N^{1} \\ \text{or} & N^{-3} & N^{-2} & N^{-1} & N^{0} \\ \text{or} & N^{-5} & N^{-4} & N^{-3} & N^{-2} \end{array}\right\} \text{putting } n = \left\{\begin{array}{c} -2 \\ -3 \\ -5 \end{array}\right.$$

&c.

Or $N^{n} \quad N^{n-1} \quad N^{n-2} \quad N^{n-3}$

may repreſent

$$\left.\begin{array}{lllll} & N^{4} & N^{3} & N^{2} & N^{1} \\ \text{or} & N^{7} & N^{6} & N^{5} & N^{4} \\ \text{or} & N^{2} & N^{1} & N^{0} & N^{-1} \end{array}\right\} \text{putting } n = \left\{\begin{array}{c} 4 \\ 7 \\ 2 \end{array}\right.$$

&c.

Or $N^{-n} \quad N^{-n-1} \quad N^{-n-2} \quad N^{-n-3}$

may repreſent

$$\left.\begin{array}{lllll} & N^{-2} & N^{-3} & N^{-4} & N^{-5} \\ \text{or} & N^{-4} & N^{-5} & N^{-6} & N^{-7} \\ \text{or} & N^{-7} & N^{-8} & N^{-9} & N^{-10} \end{array}\right\} \text{putting } n = \left\{\begin{array}{c} -2 \\ -4 \\ -7 \end{array}\right.$$

&c.

So

2. So in the Caſe of the *Coefficients.*

$$\left.\begin{array}{r} nN+N \\ nN+N+N \\ nN+N+N+N \end{array}\right\} = \left\{\begin{array}{l} \overline{n+1}N \\ \overline{n+2}N \\ \overline{n+3}N \end{array}\right.$$

&c.

Or

$$\left.\begin{array}{r} nN-N \\ nN-N-N \\ nN-N-N-N \end{array}\right\} = \left\{\begin{array}{l} \overline{n-1}N \\ \overline{n-2}N \\ \overline{n-3}N \end{array}\right.$$

And ſo it will be in caſe N be raiſed to any Power: As,

$$nN^n + N^n + N^n = \overline{n+2}\,N^n$$

Or $nN^n - N^n - N^n = \overline{n-2}\,N^n$

3. Likewiſe it is to be noted, That as (0) is an *Arithmetical Mean* between an *affirmative* and *negative* Number of equal diſtance from it, by Vertue of *Chap.* 16. *Sect.* 2. So (1) is a *Geometrical Mean* between an *affirmative* and *negative* Power of equal diſtance from it, by *Chap.* 17. *Sec.* 3. *Viz.*

$-2 \;.\; 0 \;.\; 2$ are *Arithmetical Proportionals,*
$N^{-n} \;.\; 1 \;.\; N^n$ are *Geometrical Proportionals.*

And conſequently from the ſaid *Sections* it is evident, any Power with a *negative Exponant* is equal to *unity* divided by that *Power* univerſally. For ſince,

$N^{-n} . 1 . N^n$ are *Geom. Propor.*

∴ $1 \times 1 = N^{-n} \times N^n$

∴ $N^{-n} = \frac{1 \times 1}{N^n} = \frac{1}{N^n}$ *q. e. d.*

Hence it is *obvious*, that becauſe,

$N^2 . N^1 . 1 . \frac{1}{N^1} . \frac{1}{N^2}$ are *Geomet. Prop.*

And $2 . 1 . 0 . -1 . -2$ are *Arithmet. Prop.*

and the proper *Exponants* to the ſaid *Geo. Progreſ.* it may therefore be thus expreſt; *viz.*

$$N^2 . N^1 . 1 . N^{-1} . N^{-2}$$

Or more *generally* thus; becauſe,

$N^{2n} . N^n . 1 . \frac{1}{N^n} . \frac{1}{N^{2n}}$ are *Geo. Prop.*

And $2n . n . 0 . -n . -2n$ are *Arith. Prop.*

and the *Exponants* of the ſaid *Geo. Prog.* reſpectively; therefore thus properly noted, *viz.*

$$N^{2n} . N^n . 1 . N^{-n} . N^{-2n}$$

4. Again (putting r = any *Root* to be extracted) becauſe,

$\sqrt[r]{N^{2n}} . \sqrt[r]{N^n} . 1 . \frac{1}{\sqrt[r]{N^n}} . \frac{1}{\sqrt[r]{N^{2n}}}$ are *Geo. Prop.*

And $\frac{2n}{r} . \frac{n}{r} . 0 . -\frac{n}{r} . -\frac{2n}{r}$ are *Arith. Pr.*

and the reſpective *Exponants* of the ſaid *Geo. Prog.* it may therefore be thus expreſt, *viz.*

$$N^{\frac{2n}{r}} \cdot N^{\frac{n}{r}} \cdot 1 \cdot N^{-\frac{n}{r}} \cdot N^{-\frac{2n}{r}}$$

For ſince the *Doubling*, *Tripling*, &c. of any *Exponant* is the *Squaring*, *Cubing*, &c. of the *Power*, ſo on the contrary, the taking $\frac{1}{2}$, $\frac{1}{3}$, *&c.* of the *Exponants* is the extracting the *Square*, *Cubick*, &c. Root of that *Power*: And therefore,

Corol. 1. $\sqrt[r]{N^n} = N^{\frac{n}{r}}$ or $\frac{1}{\sqrt[r]{N^n}} = N^{-\frac{n}{r}}$

2. $\frac{N^n}{\sqrt[r]{N^m}} = N^n \times \frac{1}{\sqrt[r]{N^m}} = N^{n-\frac{m}{r}}$

3. $\frac{N^n}{N^m} = N^n \times \frac{1}{N^m} = N^{n-m}$.

5. Putting $t =$ to the Number of Terms of *like Powers*; as, $N^n + N^n + N^n + N^n$, *&c.* the whole Rank will be tN^n.

But in caſe the ſaid Rank were *Multiplyed* each Term into other; as,

$N^n \times N^n \times N^n \times N^n$, *&c.*

It will be $= N^{tn}$ univerſally.

So in *Multiplying* the ſeveral Members of an *increaſing Series* into each other; as,

$N^n \times N^{n+1} \times N^{n+2} \times N^{n+3} \times$ *&c.*

The *whole Series* ſo *multiplyed* will be $= N^{\frac{2tn+tt-t}{2}}$ or from the 4° of this $= \sqrt[2]{N^{2tn+tt-t}}$ univerſally.

DEMONSTRATION.

Since the *Exponants* are $n+0$, $n+1$, $n+2$, $n+3$, *&c.* $=tn$, $+$ a *Rank of numbers* in *Arith. Prog.* whoſe firſt Term is 0, laſt $=t-1$, and common Difference $=1$, by (*Chap.* 16. *Lema* 2.) the *ſum* of ſuch Rank is $=\frac{tt-t}{2}$, but $tn+\frac{tt-t}{2}$ $=$ *ſum* of the *Exponants* in the *Series* : Therefore $\frac{2tn+tt-t}{2}=$ *Exponant* of *N* after ſuch *Multiplication.* *q. e. d.*

Again, if the *Series* were a *decreaſing* one; as, $N^{n}\times N^{n-1}\times N^{n-2}\times N^{n-3}$, *&c.* the *Exponant* of *N*, by the ſame Method of proceeding, after ſuch *Multiplication*, will be $\frac{2tn-tt+t}{2}$ or $^{2}\sqrt{N^{2tn-tt+t}}$ univerſally.

Likewiſe in caſe the *common difference* be more than *Unity*, in any ſuch *Series* put it $=$ to d : Then $\frac{2tn\pm dtt\mp dt}{2}$ will be the *Exponants* of *N* reſpectively.

6. To put this *way* of *Notation* in practice, for a *Specimen* of its Uſefulneſs and Univerſality, I ſhall firſt demonſtrate after the *common way*, how any *number N*, may be *divided* into two parts, *x* the *leſſer*, and *y* the *greater*, in any given *Ratio*, as *m* to *n*, and thoſe two *Theorems* apply to this; *viz.*

1. $m \,.\, n :: x \,.\, N-x$
2. $-mx+mN=nx$
3. $mN=nx+mx$

$\therefore\ x=\dfrac{mN}{m+n}=$ Leſſer.

But $N-x=y$

2. $N-\dfrac{mN}{m+n}=y$

3. $\dfrac{mN+nN-mN}{m+n}=y$

$\therefore\ y=\dfrac{nN}{m+n}=$ Greater. *q. e. d.*

Therefore, if we put $m+n=s$, the two *Theorems* will ſtand thus, $x=\dfrac{mN}{s}$ and $y=\dfrac{nN}{s}$, conſequently, if they be *raiſed* to any *Power* whatſoever, they may be expreſt thus, $x^n=\dfrac{m^n N^n}{s^n}$ and $y^n=\dfrac{n^n N^n}{s^n}$.

And though the Exponant (n) may be taken indefinitely, yet if it were but (3), by the *common way* they would ſtand thus;

$$xxx=\frac{mmmNNN}{mmm+3mmn+3mnn+nnn}$$

$$\text{And } yyy=\frac{nnnNNN}{mmm+3mmn+3mnn+nnn}$$

7. From the aforegoing *Two Theorems*, namely, $x = \frac{mN}{s}$ and $y = \frac{nN}{s}$, we may find x and y under ſeveral *other Conditions*, as well as in a given *Ratio*: Such as,

$$\left.\begin{array}{l} x^n = y^m \\ y^m \pm x^m = y^n \times x^n \\ y^m \pm x^m = y^n \pm x^n \\ x^n \pm x^{m-1} = y^n \\ \frac{y^m}{x^m} = y^n \pm x^n \\ \frac{x^m}{y^m} = y^n \pm x^n \\ \&c. \end{array}\right\}$$

And ſhall always be in a *Ratio* of m to n, which as well as the *Exponants* ^{m} and ^{n} may be taken *indefinitely*, only with this *Caution*, that m is always ſuppoſed *leſs* than n in both.

But then it is not every *individual* Number that is *capable* of ſuch *Diviſion*; but by this Method we have a way of finding N, or ſuch Numbers as are. For Example.

1. Query x and y; ſo that $x^n = y^m$ and $x . y$, as $m . n$. Then by the (6) of this,

	$\frac{m^n N^n}{s^n} = \frac{n^m N^m}{s^m}$
$\times\, s^n . s^m$	$s^m m^n N^n = s^n n^m N^m$
$\div\, s^m$	$m^n N^n = s^{n-m} n^m N^m$
$\div\, N^m$	$m^n N^{n-m} = s^{n-m} n^m$
$\div\, m^n$	$N^{n-m} = \frac{s^{n-m} n^m}{m^n}$
put $n - m = r$	$\therefore\ N = \sqrt[r]{\frac{s^{n-m} n^m}{m^n}}$ Univerſally.

Thus having found N, or the *number capable* of ſuch *Diviſion*, then $\frac{mN}{s} = x$, and $\frac{nN}{s} = y$, the two Numbers *ſought*, which will anſwer all the *Conditions* of the Queſtion.

Note, If it had been $tx^n = y^m$.: $N^{n-m} = \frac{s^{n-m} n^m}{tm^n}$; (*i. e.*) putting $t =$ any Number of Terms of x.

Again 2. Query x and y under theſe *Conditions* $y^m - x^m = y^n \times x^n$ and $x . y$, as m to n.

Then	$\frac{n^m N^m - m^m N^m}{s^m} = \frac{n^n N^n}{s^n} \times \frac{m^n N^n}{s^n}$
$\times s^{2n} s^m$	$s^{2n} n^m N^m - s^{2n} m^m N^m = s^m n^n m^n N^{2n}$
$\div s^m$	$s^{2n-m} n^m N^m - s^{2n-m} m^m N^m = n^n m^n N^{2n}$
$\div N^m$	$s^{2n-m} m^m - s^{2n-m} m^m = n^n m^n N^{2n-m}$
$\div n^n m^n$	$\frac{s^{2n-m} \times \overline{n^m - m^m}}{n^n m^n} = N^{2n-m}$.
put $2n-m = r$	.: $N = \sqrt[r]{\frac{s^{2n-m} \times \overline{r^m - m^m}}{n^n m^n}}$ Univerſal.

Then $\frac{mN}{s} = x$, and $\frac{nN}{s} = y$ to anſwer the ſaid Conditions.

3. Query x and y: So that $x^n + x^{n-1} = y^n$ and $x . y$ as m to n.

From which *Conditions* by the (6) of this we have

	$\frac{m^n N^n}{s^n} + \frac{m^{n-1} N^{n-1}}{s^{n-1}} = \frac{n^n N^n}{s^n}$
$\times s^n$ and s^{2n-1}	$s^{2n-1} m^n N^n + s^{2n} m^{n-1} N^{n-1} = s^{2n-1} n^n N^n$
$\div s^{2n-1}$	$m^n N^n + s m^{n-1} N^{n-1} = n^n N^n$
$\div N^{n-1}$	$m^n N + s m^{n-1} = n^n N$
Tranſp.	$n^n - m^n \times N = s m^{n-1}$
$\therefore$	$N = \frac{s m^{n-1}}{n^n - m^n}$. Univerſally.

Then $\frac{mN}{s} = x$, and $\frac{nN}{s} = y$. *q. e. d.*

It is neceſſary here to *obſerve*, That in caſe the Queſtion had been $x^n + px^{n-1} = y^n$, and p an indefinite Number, (as being the Coefficient of the ſecond Term in a Quadratick Equation) as well as the *Ratio* and *Exponants*; Then,

$$N = \frac{spm^{n-1}}{n^n - m^n}. \quad \text{Univerſally.}$$

4. Query x and y: So that $\frac{x^m}{y^m} = x^n + y^n$, and x ſhall be to y, as m to n.

by

by the (6)	$\frac{n^m N^m}{s^n}$) $\frac{m^n N^m}{s^n}$ ($=\frac{m^m}{n^n}$) $=\frac{m^n N^n + n^n N^n}{s^n}$
$\times n^n s^n$	$s^n m^m = n^m m^n N^n + n^{n+m} N^n$
∴	$N^n = \frac{s^n m^m}{n^{n+m} + n^m m^n}$
∴	$N = \sqrt[n]{\frac{s^n m^m}{n^{n+m} + n^m m^n}}$. Univerſally.

Then $\frac{mN}{s} = x$, and $\frac{nN}{s} = y$, the Two Numbers *required*; which upon Tryal will be found to Anſwer all the *Conditions* of the Queſtion.

Having given Examples *ſufficient* of the Method of raiſing *General Theorems* by this way of *Notation*, I ſhall only give ſome more *Theorems* of the Value of N, under *other Conditions* of x and y, but ſtill in a *Ratio* as m to n, and $\frac{mN}{s} = x$, and $\frac{nN}{s} = y$. *Viz.*

Conditions of x and y.	Values of N.
$y^m \pm x^n = y^n \times x^n$	$N^{2n-m} = \frac{n^m \pm m^n}{n^n m^n} s^{2n-m}$
$y^m \pm x^n = y^n \mp x^n$	$N^{n-n} = \frac{n^n \pm m^m}{n^n \mp m^n} s^{n-m}$
$x^n \pm px^{n-1} = y^n$	$N = \frac{\pm p \times m^{n-1}}{n^n m^n} s$

Con-

Conditions of *x* & *y*.	Values of *N*.
$x^n + x^n + x^n + x^n$ &c. $= y^m$ putting $t =$ number of Terms of *x*.	$N^{n-m} = \frac{n^m}{tn^n} s^{n-m}$
$x^n \times x^{n+1} \times x^{n+2}$ &c. $= y^m$ putting $t =$ number of Terms of *x*.	$N^{\frac{2m+tt-t-2m}{2}} = \frac{n^m}{m^{\frac{2m+tt-t}{2}}} s^{\frac{2m+tt-t-2m}{2}}$
$x^n \times x^{n-1} \times x^{n-2} \times x^{n-3}$ &c. $= y^m$ until $x^{n-} = 1$.	$N^{\frac{nn+n-2m}{2}} = \frac{n^m}{m^{\frac{nn+n}{2}}} s^{\frac{nn+n-2m}{2}}$
$x^n + px^{n-1} + qx^{n-2} = y^n$ and *p* and *q* indefinite, putting $\frac{spm^{n-1}}{n^n - m^n} = c$	$N = \sqrt{\frac{ssqm^{n-2}}{n^n - m^n} + \frac{cc}{4}} + \frac{c}{2}$

And putting the *Exponants* of $N = r$, and the *Theorem* $= th$; Then $N = \sqrt[r]{th}$. Univerſally.

And becauſe *m* and *n*, as well as ^{m} and ^{n}, may be taken at *pleaſure*, we may deduce *Infinite Numbers* of *particular Theorems* from each *General* one.

CHAP.

CHAP. XXI.

Of Converging Series.

THE Ingenious Mr. *Raphſon*, by his *Analiſis Equationem Univerſalis*, firſt *obliged* the World with the Method of extracting Roots by a *Converging Series*, and there exhibits

4	*particular Theorems* for all	Quadratic	*Equations.*
14		Cubic	
46		Biquadratic	

As likewiſe *Cannons* for forming *particular Theorems* for all Equations *ſimple* or *affected* to the 10th Power incluſive.

And the *Method* he takes for Raiſing theſe *Theorems* is, by ſuppoſing the *Root* in any Equation which he calls (*a*) to be a *Binomial* of *g*, a *known*, and *x* an *unknown* Quantity, and raiſes the ſaid *Binomial* to the *Power* of *a* in all the *parts* or *members* of the given Equation, and then *rejects* all the *members* of this *new* Equation, wherein there is any *Power* of *x*, and only *retains* ſuch as *conſiſts* of *g* or its *Powers*, and thoſe wherein is the *Root* or *Side* of *x*, of which he makes a *ſuppoſed* Equation, with the *abſolute* Number, and thereby finds the *ſuppoſed Value* of *x*, or the *Theorem* required: But by ſuppoſition $g + x = a$, which affords us a *new ſet* of *Theorems* for every Equation, which he there gives us likewiſe with ſeveral Examples of the Uſe of both.

Now in caſe we uſe the *laſt way* of *Notation*, all thoſe *particular Theorems*, and all others for any *Equation*, of what *degree* or *affection* ſoever, are *reduced* into one *Line* or *Univerſal Theorem*, and that by the *ſame* method of proceeding, which I ſhall now prove, and then apply in ſome particular Examples. In order whereunto put

$A =$ Abſolute Number in any Equation.
$n =$ Exponant of the higheſt Power.
$x =$ Root ſought
$N =$ Any Number taken at pleaſure.
$n =$ An unknown Number.
$1, p, q, r, s,$ &c $=$ Reſpective Coefficients of the given Equation.

Then will $N + n = x$. And

$$\pm 1 x x^{n} \pm p x x^{n-1} \pm q x x^{n-2} \pm r x x^{n-3} \pm \&c. = A.$$

repreſent any Equation whatſoever.

And becauſe $N + n$ is ſuppoſed $=$ to x, ſuch General Equation may be thus expreſt,

$$\pm 1 \times \overline{N+n}|^{n} \pm p \times \overline{N+n}|^{n-1} \pm q \times \overline{N+n}|^{n-2} \pm$$
$$\pm r \times \overline{N+n}|^{n-3} \pm \&c. = A.$$

But to bring this into a *Series*, it is neceſſary firſt to prove, That every *Power* raiſed from a *Binomial*, (without regarding the Coefficients) *conſiſts* or is *compoſed* of two *Ranks* or *Series* of *Powers*, one *increaſing* from n^{n-n} or 1 to n^{n}, and the other *decreaſing* from N^{n} to N^{n-n} or 1; and each *Member* in *one*, is multiplyed into its correſponding *Member* in the *other* reſpectively; as may appear thus,

$$\overline{N+n}^{2} = \left\{ \begin{array}{c} NN \\ 2Nn \\ nn \end{array} \right\} = \left\{ \begin{array}{l} N^{n} \times n^{n-n} \text{ or } 1 \\ N^{n-1} \times n^{n-1} \text{ Twice} \\ N^{n-n} \times n^{n} \end{array} \right.$$

Again,

$$\overline{N+n}^{3} = \left\{ \begin{array}{c} NNN \\ 3NNn \\ 3Nnn \\ nnn \end{array} \right\} = \left\{ \begin{array}{l} N^{n} \times n^{n-n} \\ \left.\begin{array}{l} N^{n-1} \times n^{n-2} \\ N^{n-2} \times n^{n-1} \end{array}\right\} \text{Thrice} \\ N^{n-n} \times n^{n} \end{array} \right.$$

And ſo it will always be *ad Infinitum.* *q. e. d.* Hence theſe Two *Corolaries.*

(1.) That the *Coefficient* of the *ſecond Term* in any *Power* raiſed from a *Binomial*, is always $=n$ the *Exponant* of the *higheſt Power.*

(2. That the *Root* or *Side* of n^n the unknown quantity, is always *multiplyed* into the *ſecond Term* of the known.

Now from the *latter*, it is evident we are (in this caſe) but to make *uſe* of the two *firſt Members* of the Power of ſuch *Binomial*; and by the *firſt* we may expreſs the *Coefficient* of the *ſecond Term*, by n the *Exponant* of the *Power*: Therefore the former *Equation* will now ſtand thus;

$$\pm 1 \times \overline{N^{n} \pm {}_{n}N^{n-1}n} \pm p \times \overline{N^{n-1} \pm {}_{n-1}N^{n-2}n} \pm q \times \overline{N^{n-2} \pm {}_{n-2}N^{n-3}n} \pm r \times \overline{N^{n-3} \pm {}_{n-3}N^{n-4}n} \pm \&c. = A.$$

Now to find the *Value* of *n*, or the unknown Quantity, *It is plain*, that thoſe *Members* into which it is *multiplyed*, will be the *Diviſor* with the *ſame* Signs, and the *others* the *Dividend*, but with *contrary* Signs, as being to be *tranſpoſed* to the other ſide of the *Equation*: Therefore firſt,

$$n = \frac{A \pm 1 \times N^{n} \pm p \times N^{n-1} \pm q \times N^{n-2} \pm}{1 \times {}_{n}N^{n-1} \pm p \times {}_{n-1}N^{n-2} \pm q \times {}_{n-2}N^{n-3} \pm}$$

$$\frac{\pm r \times N^{n-3} \quad \&c.}{\pm r \times {}_{n-3}N^{n-4} \, \&c.}$$

Which Theorem *exhibits* all poſſible *particular ones*, for extracting of Roots, according to the *firſt ſort of Mr.* Ralphſon's, agreeing exactly with them, as will be found on Tryal, always remembring that the *Signs* in the Dividend muſt be *contrary* to thoſe in the Equation, and in the Diviſor the *ſame* reſpectively.

But $N + n = x$: Therefore ſecondly,

$$x = \frac{A \pm 1 \times_{n-1} N^{n} \pm p \times_{n-2} N^{n-1} \pm q \times_{n-3} N^{n-2} \pm}{1 \times {}_{n}N^{n-1} \pm p \times_{n-1} N^{n-2} \pm q \times_{n-2} N^{n-3} \pm}$$

$$\frac{\pm r \times_{n-4} N^{n-3} \quad \&c.}{\pm r \times_{n-3} N^{n-4} \quad \&c.}$$

Which *gives* us all thoſe of the *ſecond ſort* univerſally: but in this Caſe the *Signs* both in the Dividend and Diviſor will be the *ſame* as in the given Equation reſpectively: As likewiſe it may be proper to *take Notice*, That if any Term be *wanting* in the Equation, the ſame muſt be *omitted* in *either* Theorem reſpectively.

Now from either of theſe *Two Generals*, to deduce any *particular Theorem*, for finding the *Root* of any *given Equation*, we need only conſider, that $\Lambda^{n-n} = 1$ or $\frac{\Lambda^n}{\Lambda^n} = 1$, that *Unity* will neither Multiply nor Divide; as alſo that ${}_n - {}_nN^n = 0$, or ${}_nN^n - {}_nN^n = 0$, and any *Quantity* multiplyed into 0, is $= 0$, and when either Caſe happens, (which always will, except where the laſt Term is wanting) the *Theorem* is determin'd. Therefore,

Then we ſhall find $n =$ … $x =$

Suppoſe any Equ as,	By the 1ſt Gen. Th.	By the 2d Gen. Th.
$x^2 = A$	$\frac{A - N^2}{{}_2N}$	$\frac{A + N^2}{{}_2N}$
$x^3 = A.$	$\frac{A - N^3}{{}_3N^2}$	$\frac{A + {}_2N^3}{{}_3N^2}$
$x^4 = A$	$\frac{A - N^4}{{}_4N^3}$	$\frac{A + {}_3N^4}{{}_4N^3}$
$x^2 \pm px = A$	$\frac{A - N^2 \mp pN}{{}_2N \pm p}$	$\frac{A + N^2}{{}_2N \pm p}$
$-x^2 \pm px = A$	$\frac{A + N^2 - pN}{-{}_2N + p}$	$\frac{A - N^2}{-{}_2N + p}$
$x^4 \pm px^3 = A$	$\frac{A - N^4 \mp pN^3}{{}_4N^3 \pm {}_3pN^2}$	$\frac{A + {}_3N^4 \pm {}_2pN^3}{{}_4N^3 \pm {}_3pN^2}$
$x^3 \pm px^2 \pm qx = A$	$\frac{A - N^3 \mp pN^2 \mp qN}{{}_3N^2 \pm {}_2pN \pm q}$	$\frac{A + {}_3N^3 \pm pN^2}{{}_3N^2 \pm {}_2pN \pm q}$
&c.		

And after the ſame manner for any *Equation* whatſoever.

Thus having the *particular* Theorem, the *Application* in either Caſe are as follows;

Let N be any *number* taken at pleaſure, as before.

T be = *Theorem*, in which N muſt always be of its laſt Value found.

Then the *Proceſs* will be of the

Firſt General Theorem.	Second Gen. Theor.
N the 1ſt $\pm T = N$ the 2d Then	$T = N$ the 2d Then
N the 2d $\pm T = N$ the 3d Then	$T = N$ the 3d Then
N the 3d $\pm T = N$ the 4th Then	$T = N$ the 4th Then
N the 4th $\pm T = N$ the 5th, &c.	$T = N$ the 5th, &c.

Some of which *Values* of N will *terminate* in the *true* Root ſought if it have one; but if it be a *Surd*, then the *Value* of N will proceed into an *Infinite Series*, but may be proſecuted nearer the Truth than any aſſignable; which *Series*, each Operation, will proceed in *number of places*, in a *Geometrical Progreſſion*, whoſe firſt Term is 1, and Ratio = 2. *Viz.* Firſt 1, then 2, then 4, then 8, then 16, then 32, &c. places.

It is likewiſe obſervable, That the *firſt* General Theorem *Converges*, by finding out a *number* to be added to, or ſubtracted from the *laſt Value* of N, (as it ſhall be affected with + or —,) until N be $= x$ ſought. So the *laſt*, *Converges* by N it ſelf, whoſe *Value* each Operation, ſhall grow nearer and nearer, until it be = to x ſought.

We may alſo take Notice, That though N be *aſſumed* never ſo far from the *true* Root, yet it will Converge to it, by renewing the Operation. But the Work may be much ſhortened, in caſe we *point* the given Equation (if it will admit of it) both in the *abſolute Number* and *Coefficients*, according to their reſpective *degrees* of *Adfection*: And take firſt 1, then 2, then 4, &c. of thoſe *Points*, (from the firſt) each Operation; For it is evident; the Coefficients *increaſe* in their Powers, as the higheſt unknown Term *decreaſes*; therefore the *abſolute Number*, is of the ſame *Power* with the higheſt unknown *Quantity*.

One Inſtance may be ſufficient to *explain* it, and therefore ſuppoſe this *Cubic* Equation were to be pointed;

Viz. $xxx + 25xx + 836x = 53297$

Or $xxx + pxx + qx = A$

Then it would be $xxx + \dot{2}\dot{5}xx + \dot{8}3\dot{6}x = 5\dot{3}29\dot{7}$

For the *Abſolute* Number is a Cube ⎫ and are
Coefficient q a Square ⎬ *pointed* ac-
p a Lateral ⎭ cordingly.

And the like Method for any other Equation, where it will admit of it.

Now to apply this laſt, we are to take
the *Firſt* Operation $xxx + 2xx + 8x = 53$
Second Operation $xxx + 25xx + 836x = 53297$

And conſequently the *Value* of the Coefficients as well as the Abſolute Number *alters*, ſo long as there are *Punctations*.

But by a few *Numerical* Operations, the ſaid Notifications, as well as the *Method* of the Proceſs of *each Theorem*, will be further illuſtrated: Therefore,

1. Suppoſe $xx = 2 = A$. Query x by the firſt General Theorem.

Then $n = \frac{A-N^2}{2N} = T$, and take $N = 1$

Therefore $1 + T\,(= ,5) = 1,5 = N$ the 2d.
∴ $1,5 - T\,(= -,083) = 1,417 = N$ the 3d.
∴ $1,417 - T\,(= -,002783) = 1,414217 = N$ 4th
∴ $1,414217 - T\,(= -,0000003437622) =$
$= 1,414213562378 = N$ the 5th $= x$.

2. Suppoſe $xx = 2 = A$. Query x by the ſecond General Theorem.

Then $x = \frac{A+N^2}{2N} = T$, and take $N = 1$, as before.

Therefore $T = 1,5 = N$ the 2d.
∴ $T = 1,416 = N$ the 3d.
∴ $T = 1,414215 = N$ the 4th.
∴ $T = 1,41421356237 3 = N$ the 5th, $= x$.

By which it is evident; Firſt, That both Theorems amount to the *ſame thing*, the difference being only in the laſt Figure, which would be corrected the next Operation. Secondly, That x will proceed into an *Infinite Series*, if a Surd. Thirdly, That each Operation gives *double* the Number of Figures of the laſt.

3. Suppoſe $xxxx = 2839,8241 = A$. Query x by *Theor.* 1.

Then $n = \frac{A - N^4}{4N^3} = T$, and take $N = 10$.

Therefore $10 - T\ (= -3) = 7 = N$ the 2d.
∴ $7 + T\ (= +,4) = 7,4 = N$ the 3d.
∴ $7 - T\ (= -,1) = 7,3 = x$, the true Biquadratic Root ſought.

4. Suppoſe $xxxx = 2839,8241 = A$, as before. Query x by the ſecond Theorem.

Then $x = \frac{A + 3N^4}{4N^3} = T$, and take $N = 5$.

Therefore $T = 5,6 = N$ the 2d.
∴ $T = 8,2 = N$ the 3d.
∴ $T = 7,4 = N$ the 4th.
∴ $T = 7,3 = N$ the 5th $= x =$ true Root ſo.

From which two laſt Examples it appears; Firſt, *Either* Theorem will find the *true Root*, if it have one. Secondly, That it *matters not*, whether N be taken *above* or *below* the Root, nor how *far* from it.

5. Suppoſe $xx + 587x = 987459$ or $xx + px = A$. Query x by the 1. Theor. (*i. e.*) $n = \frac{A - N^2 - pN}{2N + p} = T$.

Becauſe of the Punctations we are to take,

1. Operation $xx + 5x = 98$
2. $xx + 58x = 9874$
3. $xx + 587x = 987459$

And ſuppoſe $N = 8$.

Therefore $8 - T\ (= -{,}2) = 78 = N$ the 2d.

∴ $78 - T\ (= -3{,}4) = 74{,}6 = N$ the 3d.

∴ $74{,}6 - T\ (= -3{,}34) = 742{,}66 = N$ the 4th

∴ $742{,}66 - T\ (= -{,}012689) = 742{,}647311$
$= x$ ſought.

6. Suppoſe $xx - 20x = 53482$ or $xx - px = A$. Query x by the ſecond General Theorem.

Then $x = \frac{A + N^2}{2N - p} = T$, and take $N = 250$.

Therefore $T = 241 = N$ the 2d.

∴ $T = 241{,}4 = N$ the 3d.

∴ $T = 241{,}475 = N$ the 4th.

∴ $T = 241{,}477860 = N$ the 5th, $= x$ ſought

From theſe two laſt it is plain; Firſt, That there is no *abſolute neceſſity* for Punctation. Secondly, That Punctation does nevertheleſs *ſhorten* the Work, where it can be done.

But I hope I have ſaid enough to make the *whole matter*, as well as the *manner*, of proceeding *plain* and *eaſie*, to the meaneſt Capacity; and though I have given *Numerical* Examples, no farther than an *affected* Quadratic, yet 'tis the ſame to any *degree* of Power or Affection whatſoever, regard being had to its *proper* and *particular* Theorem, deduced from either of the *general* ones.

CHAP.

CHAP. XXII.

Theorems Arithmetical.

1. PUT $a =$ a greater Number, and $b =$ to a leſſer.

Then, in *Addition* $a + b =$ Sum.
Subſtraction $a - b =$ Difference.
Multiplication $ab =$ Product.
Diviſion $\frac{a}{b} =$ Quote.

II. *Of Fractions.*

Put $n =$ Numerator, $d =$ Denominator, $c =$ Common Meaſure, $i =$ Integer, $f =$ a Fraction.

Then to reduce a f into its leaſt Terms $\frac{n}{c} + \frac{d}{c} = \frac{n+d}{c}$

i into f of a given d $\frac{i \times d}{d} = \frac{id}{d}$

$i+f$ into an Improp. f $\frac{i \times d+n}{d} = \frac{id+n}{d}$

an Improper f $d)\,id+n\,(i+\frac{n}{d}$

f's of div. d's into 1 d $\frac{N}{D}\ \frac{n}{d} = \frac{Nd}{Dd}\ \frac{nD}{Dd}$

$$\left.\begin{array}{l}\textit{Addition,}\\ \\ \textit{Subtraction.}\end{array}\right\}\begin{array}{c}\text{Firſt reduce}\\ \text{them into}\\ \text{one } d\text{, then}\end{array}\left\{\begin{array}{l}\frac{N+n}{d} = \text{Sum.}\\ \\ \frac{N-n}{d} = \text{Difference.}\end{array}\right.$$

Multiplication of Integ. & Fract. $\frac{i\times n}{d}=\frac{n}{i)d}=$ Product.

both Fractions $\frac{N\times n}{D\times d}=\frac{Nn}{Dd}=$ Product

$$\left.\begin{array}{ll}\textit{Diviſion} \text{ of } f \text{ by } i \text{ ——} & \frac{n}{i\times d}=\frac{n}{id}\\ \text{of } i \text{ by } f \text{ ——} & \frac{i\times d}{n}=\frac{id}{n}\\ \text{of } f \text{ by } f \text{ ——} & \frac{N}{D}\Big)\frac{n}{d}=\frac{Dn}{Nd}\\ \text{of equal } d\text{'s ——} & \frac{N}{D}\Big)\frac{n}{D}=\frac{n}{N}\end{array}\right\}\text{Quotes.}$$

Vulgar into Decimals —— $\frac{n}{d}$.

III. *Reduction.*

Put the given Number of the greater Denomination $= N$, and the Number of Unites of the leſſer Denomination which is = to one in the greater $= n$.

Of a Greater into a Leſſer, or Deſcending —— Nn.

Leſſer into a Greater, or Aſcending —— $\frac{N}{n}$.

IV. *Rule*

IV. *Rule of Three Single.*

There being three Terms always given, to find a fourth; *viz.*

$$\left.\begin{array}{l}\text{2 of one Denom.} = a, \& \ c \\ \text{1 of another} \text{ — } b,\end{array}\right\} \text{to be placed} \left\{\begin{array}{c} a\\ b\\ c\end{array}\right\} \text{in the} \left\{\begin{array}{c}1\\ 2\\ 3\end{array}\right\} \text{place,}$$

and put the Number sought $= x$.

$$\text{Then in Direct} \text{———} x = \frac{bc}{a}.$$

$$\text{Indirect} \text{———} x = \frac{ab}{c}.$$

Ditto Double.

Where five Terms are always given, to find a sixth; *viz.*

$$\left.\begin{array}{ll}\text{2 of one Denom.} & = a \ \& \ d \\ \text{2 of another} & = b \ \& \ e \\ \text{1 of another} & = c\end{array}\right\} \text{to be plac'd} \left\{\begin{array}{c} a\\ b\\ c\\ d\\ e\end{array}\right\} \text{in the} \left\{\begin{array}{c}1\\ 2\\ 3\\ 4\\ 5\end{array}\right\} \text{place,}$$

and put the Term sought $= x$.

$$\text{Then in Direct} \text{———} x = \frac{cde}{ab}.$$

$$\text{Indirect} \text{———} x = \frac{bcd}{ae}.$$

V. *Rule of Fellowſhip, without Time.*

Put A's Stock $= a$. $\therefore$ his Share of the Gain or Loſs $= \frac{ad}{a+b+c}$.

B's $= b$. $\therefore$ ditto $= \frac{bd}{a+b+c}$.

C's $= c$. $\therefore$ ditto $= \frac{cd}{a+b+c}$.

And the whole Gain or Loſs $\Big\} = d$.

Ditto with Time.

Put A's Time $= p$. $\therefore$ his Share $= \frac{apd}{ap+bq+cr}$.

B's $= q$. $\therefore$ ditto $= \frac{bqd}{ap+bq+cr}$.

C's $= r$. $\therefore$ ditto $= \frac{crd}{ap+bq+cr}$.

Their Stocks, and the whole Loſs or Gain as before.

VI. *Rule of Falſe, Single Poſition.*

Put $n =$ any ſuppoſititious Number, $a =$ abſolute Number given, $N =$ to the Sum produced by n, when ordered according to the Queſtion, and $x =$ true Number ſought.

Then $\frac{an}{N} = x$.

Ditto

Ditto Double Poſition.

Put m and n = to two ſuppoſititious Numbers, p and q = to their reſpective Errors, when ordered according to the Conditions of the Queſtion, and a and x as before :

$$\text{Then if } \left.\begin{matrix} +p+q \\ \text{or } -p-q \end{matrix}\right\} \ldots x = \frac{mq-np}{q-p} . \quad \text{If } \left.\begin{matrix} +p-q \\ \text{or } -p+q \end{matrix}\right\} \ldots x = \frac{mq+np}{q+p} .$$

VII. *Rule of Alligation, Medial.*

Put s = ſum of the Quantities to be mixt, v = total Value of the Species, p = any part of the mixture, and x = mean Rate required of that part :

$$\text{Then } \ldots . \frac{vp}{s} = x.$$

Ditto Alternate Partial.

Put a, b, c, d, &c. = reſpective Rates of Quantities to be mixt, p, q, r, s, &c. = reſpective alternate Differences, alligated with the mean price, m = Quantity of one of them given, and x, y, z, &c. the reſpective Quantities ſought.

$$\text{Then } \frac{qm}{p} = x . \frac{rm}{p} = y . \frac{sm}{p} = z . \text{ \&c.}$$

Ditto Alternate Total.

Put a, b, c, d, &c. = reſpective Quantities of things to be mixt, p, q, r, s, &c. = reſpective alternate Differences of the ſeveral Rates alligated with the mean price, and x, y, z, u, &c. the reſpective quantities of each, to be afforded at that mean price, ſought.

$$\text{Then } \frac{ap+bp+cp+dp}{p+q+r+s}=x. \quad \frac{aq+bq+cq+dq}{p+q+r+s}=y$$

$$\frac{ar+br+cr+dr}{p+q+r+s}=z. \quad \frac{as+bs+cs+ds}{p+q+r+s}=v.$$

THEOREMS Numerical.

1. TO divide $N=$ any given Number, into x and y, in any given difference $=d$.

Then $\frac{N-d}{2}=x=$ leſſer, and $\frac{N+d}{2}=y=$ greater.

2. Ditto in any given Ratio as m to n.

Then $\frac{mN}{m+n}=x$, and $\frac{nN}{m+n}=y$.

3. Query a Number N, which may be divided into x and y in any given difference (d), ſo that m times y, will be $=$ to n times x.

$$\text{Then} \ldots \frac{n+m}{n-m} \cdot d = N.$$

Ditto that $m^m y = n^n x$.: $\frac{n^n+m^m}{n^n-m^m} d = N$.

4. To divide N into mean and extream Reaſon. Put the greater part $= x$.

$$\text{Then } x = \sqrt{\frac{5NN}{4}} - \frac{N}{2}.$$

5. Query $x, y, z,$ and v: So that $xyz = a$, $yzv = b$, $zvx = c$, and $vxy = d$.

$$\therefore z = \sqrt[3]{\frac{abc}{dd}} \,.\; y = \sqrt[3]{\frac{abd}{cc}} \,.\; x = \sqrt[3]{\frac{acd}{bb}} \,.\; v = \sqrt[3]{\frac{bcd}{aa}}$$

6. Query $x, y,$ and z: So that $xz + yz = a$, $yx + zx = b$, and $yx + yz = c$.

$$\text{Put } \frac{-a+b+c}{2} = d \,.\; \frac{a-b+c}{2} = e \text{ and } \frac{a+b-c}{2} = f$$

$$\text{Then will } x = \sqrt{\frac{df}{e}} \,.\; y = \sqrt{\frac{de}{f}} \,.\; z = \sqrt{\frac{ef}{d}}.$$

7. To divide dd into two Squares: Take any two Numbers $r =$ leſſer, and $s =$ greater:

Then $\frac{2rsd}{rr+ss} =$ firſt Square, and $\frac{ssd-rrd}{ss+rr} =$ ſecond.

8. To find the Unciæ of any Power raiſed from a Binomial: Put $n =$ Exponant of the Power.

Then $1 \times \frac{n-0}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times$ &c. $=$ Num. Unciæ.

Note, This and all other *Theorems* of this kind, may be continued at pleaſure.

9. To *Extract* any *Root* of a *Binomial* or *Reſidual*, &c. or to form any Power from it.

Put F = firſt Member, L = laſt or Remainder, and $\frac{L}{F} = q$, $\frac{m}{n}$ = Exponent of the requir'd Root or power, x = Root or power ſought.

Likewiſe put $\overline{F}|^{\frac{m}{n}} = a$, $\frac{m}{n}q = b$, $\frac{m-n}{2n}q = c$, $\frac{m-2n}{3n}q = d$, $\frac{m-3n}{4n}q = e$, &c.

Then $x = a + ab + abc + abcd + abcde +$ &c.

10. Query x, y, and z. So that $x + y = a$, $y + z = b$, and $z + x = c$.

Then $x = \frac{a-b+c}{2}$, $y = \frac{a+b-c}{2}$ and $z = \frac{-a+b+c}{2}$.

11. Query x, y, and z, in *Geometrical Progreſſion*: So that $xy = a$ and $yz = b$.

Then $x = \sqrt[4]{\frac{aaa}{b}}$, $y = \frac{a}{x}$ and $z = \frac{b}{y}$.

12. Query x, y, z and u, in *Geometrical Progreſſion*: So that $xy = a$ and $zu = b$.

Then $x = \sqrt[8]{\frac{a^5}{b}}$, $y = \frac{a}{x}$, $z = \frac{a^2}{x^3}$ and $v = \frac{b}{z}$.

13. Query x, y and z, in *Geometrical Progreſſion*: So that $x + y$ and $y + z$ ſhall be *Rational Squares*. Take a and b = any Two Numbers;

Then $y = \frac{aabb}{aa+bb}$, $x = aa - y$ and $z = bb - y$.

14. Query x and y in *Rational Numbers*: So that $xy+x$ or $xy+y$ ſhall be *Rational Cubes*, take $b=$ any Rational Fraction;

$$\text{Then } x=\frac{2b-1}{b^3+3b^2} \text{ and } y=b^3x^2-1.$$

Of Simple Intereſt.

(1.) For Money paid before or after 'tis due.

Put $a=$ Principal or Sum forborn.
$t=$ Time of Forbearance in years or parts.
$r=$ Gain or Rate of 1 *l.* in one Year.
$u=$ Amount.

$$\textit{Data, } a, r, t, \textit{ Quiſita} \text{——} u=\overline{rt+1}\times a.$$

$$u, r, t, \text{————} a=\frac{u}{rt+1}$$

$$a, u, r, \text{————} t=\frac{u-a}{ar}$$

$$a, t, u, \text{————} r=\frac{u-a}{at}$$

(2.) Of many equal Payments at many equal times.

Put $u=$ Annuity or Penſion.
$t=$ Time of continuance in years or parts.
$r=$ Gain or Rate of 1 *l.* for one Year.
$p=$ Price or preſent Worth.

Data $u, t, r,$ *Quæſita* $p = \frac{\overline{rtt + 2t - rt} \times u}{2rt + 2}$

$p, t, r,$ ——— $u = \frac{\overline{2rt + 2} \times p}{rtt + 2t - rt}$

$u, t, p,$ ——— $r = \frac{\overline{ut - p} \times 2}{2pt + ut - utt}$

$u, r, p,$ ——— $t = \frac{\left.\begin{matrix} +2pr \\ +ur \\ -2u \end{matrix}\right\} + \sqrt{\left\{\begin{matrix} +2pr \\ +ur \\ -2u \end{matrix}\right\}^{2} + 8pur}}{2ur}$

Of Compound Intereſt.

(3.) Money paid before or after due.

Put a = Principal or Sum forborn.
t = Time of Forbearance in Years, half Years, Quarters or Days.
r = Rate of Intereſt *per Cent.* by ditto.
u = Amount.

Data, $a, r, t,$ *Quæſita* —— $u = r^{t} \times a$

$u, r, t,$ ——— $a = \frac{u}{r^{t}}$

$a, r, u,$ ——— $t = r \div) \frac{u}{a}$

$a, u, t,$ ——— $r = \sqrt[t]{\frac{u}{a}}$

(4.) An-

(4.) Annuity or Penſions.

Put u = Annuity or Penſion.
t = Time of Continuance.
r = Rate of Intereſt.
p = Price or preſent Worth.

Data, u, t, r, *Quiſita* $p = \frac{u - \frac{u}{r^t}}{r-1}$

$p, t, r,$ ——— $u = \frac{r^t \times \overline{r-1} \times p}{r^t - 1}$

$p, r, u,$ ——— $t = \frac{r^t}{r} = \frac{u}{p+u-pr}$

$u, t, p,$ ——— $r = \frac{p+u}{p} - \frac{u}{r^t \times p}$

(5.) Of Buying and Selling Eſtates, Annuities, Penſions, &c.

Put u = Annuity or Penſion.
r = Rate allowed for Intereſt.
p = Price or preſent Worth.

Data, p, r, *Quiſita* —— $u = p \times \overline{r-1}$

$u, r,$ ——— $p = \frac{u}{r-1}$

$p, u,$ ——— $r = \frac{p+u}{p}$

(6.) How many Years Purchaſe an Annuity may be worth.

Put y = Number of Years.
r = Rate allowed for Intereſt.

Data, r, *Quiſita* —— $y = \frac{1}{r-1}$

y —————— $r = \frac{y+1}{y}$

Of Logarithms,

Or a *Rank of Numbers* in *Arithmetical Progreſſion*, accommodated to a *Rank of Numbers* in *Geometrical Progreſſion.*

Put A, B, *&c.* = any *Numbers*, and a, b, *&c.* the reſpective *Logarithm* of thoſe Numbers, p = Number of *Integer places* in any Number, and c = *Characteriſtic* of the *Logarithm* of that Number.

Then $c = p - 1$, and $p = c + 1$.

So $a + b = A \times B$ and $a - b = \frac{A}{B}$

Likewiſe $\frac{a}{2} = \sqrt{A}$, $\frac{a}{3} = \sqrt[3]{A}$. $\frac{a}{4} = \sqrt[4]{A}$. &c.

And $2a = A^2$. $3a = A^3$. $4a = A^4$. &c.

Again, put $m =$ *Mean Proportional*, $n =$ any *Number* of *Mean Proportionals* required, and put $\frac{a-b}{n+1} = q$: Then $\frac{a+b}{2} = \sqrt{AB} = m$.

$$\left.\begin{array}{l}\text{So } b+q = \text{1ſt} \quad b+2q = \text{2d} \quad b+3q = \text{3d, \&c.} \\ \text{Or } a-q = \text{1ſt} \quad a-2q = \text{2d} \quad a-3q = \text{3d, \&c.}\end{array}\right\} = nm.$$

Again, Let $\frac{A}{B} =$ *Fraction*, Then $a - b =$ Log. of that Fraction, and put $p =$ place from Unity, where the firſt Figure in the *Decimal Fraction* ſhall ſtand; Then will $10 - p = c$ or $c = p - 10$, and are to be ſo uſed in *Addition* or *Subſtraction* of Log.

For in Addition of *Integers* and *Decimal parts*, put $s =$ *Sum* of the Characteriſtics.

$$\text{Then } s \text{, if} \left\{\begin{array}{l}\text{above 10, reject 10, the } Rem. \text{ will be} = \text{Int.} \\ \text{under 10} \qquad\qquad \text{the } Sum \text{ will be} = \text{parts}\end{array}\right.$$

Of *Parts* by *Parts*.

$$s\text{, if} \left\{\begin{array}{l}\text{under 10, add 10.} \;\therefore\; Sum \\ \text{above 10, reject 10.} \;\therefore\; Rem.\end{array}\right\} = \text{parts.}$$

In Subſtraction, *either* or *both Parts*, If the *higher* Char. be a *ſmaller* Fig. than the *lower*, add 10 to it, and obſerve whether it be of *greater Value*; if ſo, the *Remainder* will be *Integers*; if not, *Parts*.

But in Multiplication and Diviſion of the Log. of *Fractions*, by an Integer Number x, 'twill be more ready, to put c, of the *Log*. of ſuch Fraction $= -p$; $\therefore p = -c$.

Then in Multiplication, put the *Number* to be *carried*, from the *Mult*. of the *Log*. to the Char. $= t$, $x \times -c$, $(= -xc) - t =$ New Charact.

In Diviſion, add ſo many *Units* to the *Characteriſtic* until it can be *divided*, which call t: Then $\frac{-c+t}{x}$ = New c, and *carry* ſo many 10^s as t is, to the firſt Fig. of the *Log.*

The *Arithmetical Complement* of any *Log.* is the *remainder* of every Figure (including the Char.) to 9, except the laſt *ſignificant Fig.* to the Right-hand, whoſe *remainder* muſt be to 10.

Put *Arith.Co.* of the *Log.* of $B = z$: Then $a + z = \frac{A}{B}$.

Of making Logarithms.

Put $\frac{1}{m}$ = *Index* of the *Power* whoſe *Root* is ſought, q = any Number + or − than 1: Then by the 8th and 9th of this, ſuppoſing m infinite.

$$\overline{1+q}|^{\frac{1}{m}} = 1 + \frac{1}{m}q - \frac{1}{2m}qq + \frac{1}{3m}qqq \text{ \&c.}$$

∴ $\frac{1}{m}$ into $q - \frac{1}{2}qq + \frac{1}{3}qqq$, &c. is the *Augment* of the firſt *Mean Proportional* between 1, and $1+q$, and is therefore the *Log.* of the *Ratio* of 1 to $1+q$. Alſo,

$$\overline{1-q}|^{\frac{1}{m}} = 1 - \frac{1}{m}q - \frac{1}{2m}qq - \frac{1}{3m}qqq \text{ \&c.}$$

∴ $\frac{1}{m}$ into $q - \frac{1}{2}qq - \frac{1}{3}qqq$, &c. is the *Decrement* of the firſt *Mean Proportional* between 1 and $1-q$, and is therefore the *Log.* of the *Ratio* of 1 to $1-q$.

Again, put the Terms of any *Ratio* $= r$ and s: $\therefore q = \frac{s-r}{r}$ in an increaſing, and $q = \frac{s-r}{s}$ in a decreaſing *Ratio*: Alſo put $r+s=z$, and $r-s=d$. Then will

$$\left.\begin{array}{l}\frac{1}{m}\text{ into }\frac{d}{r}-\frac{d^2}{2r^2}+\frac{d^3}{3r^3}-\frac{d^4}{4r^4}\ \&c.\\ \text{Or }\frac{1}{m}\text{ into }\frac{d}{s}+\frac{d^2}{2s^2}+\frac{d^3}{3s^3}+\frac{d^4}{4s^4}\ \&c.\end{array}\right\}=Log.\text{ of }\frac{s}{r}.$$

So putting $\frac{1}{2}z = \frac{1}{2}r + \frac{1}{2}s$, we have

$$\frac{1}{m}\text{ in }\frac{d}{z}+\frac{d^2}{2z^2}+\frac{d^3}{3z^3}+\frac{d^4}{4z^4}\ \&c. = Log.\text{ of }\frac{\frac{1}{2}z}{r}$$

$$\text{And }\frac{1}{m}\text{ in }\frac{d}{z}-\frac{d^2}{2z^2}+\frac{d^3}{3z^3}-\frac{d^4}{4z^4}\ \&c. = Log.\text{ of }\frac{s}{\frac{1}{2}z}$$

And by adding theſe two *Log.* together, we have

$$\frac{1}{m}\text{ in }\frac{2d}{z} * + \frac{2d^3}{3z^3} * + \frac{2d^5}{5z^5}\ \&c. = Log.\text{ of }\frac{\frac{1}{2}z}{r}\times\frac{s}{\frac{1}{2}z}=\frac{s}{r}$$

$$\therefore\ \frac{1}{m}\text{ in }\frac{2d^2}{2z^2} * + \frac{2d^4}{4z^4} * + \frac{2d^6}{6z^6}\ \&c. = Log.\text{ of }\frac{r)\frac{1}{2}z}{\frac{1}{2}z)s}$$

$\left(=\frac{\frac{1}{4}zz}{rs}\right)=\frac{\frac{1}{2}z}{\sqrt{rs}}\times\frac{\frac{1}{2}z}{\sqrt{rs}}$. And half this *Log. Viz.*

$$\frac{1}{m}\text{ in }\frac{d^2}{2z^2} * + \frac{d^4}{4z^4} * + \frac{d^6}{6z^6}\ \&c. = Log.\text{ of }\frac{\frac{1}{2}z}{\sqrt{rs}}$$

(*i. e.*) of the *Ratio* of *Geom.* to the *Arith. Mean* between r and s.

Again, putting $\frac{1}{4}zz + rs = y$: Then ſince $y = z$ and $1 = x$ having the *Log.* of the *Number* next above and below any prime *Number*, we may diſcover that likewiſe

For $\frac{1}{m}$ in $\frac{2}{y} + \frac{2}{3y^3} + \frac{2}{5y^5} + \frac{2}{7y^7}$ &c. $=$ *Log.* of $\frac{\frac{1}{4}zz}{rs}$

∴ $\frac{1}{m}$ in $\frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + \frac{1}{7y^7}$ &c. $=$ *Log.* of $\frac{\frac{1}{2}z}{\sqrt{rs}}$.

Note, When m is taken 10000 &c. it may be neglected, and *Napier*'s *Log.* will enſue: But if you would have *Briggs*'s, Divide

0,43429448190325182765112891891660508229439700580366566114454, by the *Powers* of z and y continually, according to the *Theorem*, reſerving the *Quotes* to be added together.

THEOREMS *Geometrical.*

1. OF *Lines*; put the whole Line $= l$, the greater part of it $= m$, and the leſſer $= n$: Then will $m + n = l$, $m - n = d$, $l - n = m$, and $l - m = n$, and put $a =$ to another Line, and we have,

1. $al = am + an$.
2. $ll = lm + ln$.
3. $lm = mm + mn$, or $ln = nn + mn$.
4. $ll = mm + 2mn + nn$.
5. $\frac{ll}{4} = mn + \frac{dd}{4}$.
6. $ld + nn = mm$.
7. $2ln + mm = ll + nn$, or $2lm + nn = ll + mm$.
8. $4ln + mm = ll + 2ln + nn$.
Or $4lm + mm = ll + 2lm + mm$.
9. $\frac{ll + dd}{2} = mm + nn$, or $ll + dd = 2mm + 2nn$.

2. In a *Right-angled Triangle* put $b =$ Baſe, $p =$ Perpendicular, and $h =$ Hypotenuſe: Then,

$$hh = pp + bb \quad .\colon \quad h = \sqrt{pp + bb}.$$

$$pp = hh - bb \quad .\colon \quad p = \sqrt{hh - bb}.$$

$$bb = hh - pp \quad .\colon \quad b = \sqrt{hh - pp}.$$

3. In an *Obtuſe-Angled Triangle*, put $b =$ whole Baſe, $a =$ greater ſide, $c =$ leſſer, $y =$ greater Segment of the Baſe, and $x =$ leſſer:

Then $y = \dfrac{bb + aa - cc}{2b}$ and $x = \dfrac{bb + cc - aa}{2b}$.

4. In an *Acu'e-Angled Triangle*, put the $b =$ Baſe, $a =$ greater ſide, $c =$ leſſer, and $x =$ Segment of the Baſe, to compleat it to a *Right-Angled Triangle*: Then $x = \dfrac{aa - cc - bb}{2b}$.

II. *Of Superficial Meaſure.*

1. In all *Squares, Parallelograms, Rombs, Romboids*, &c. put $l =$ length, $b =$ breadth, and $x =$ Area: Then $x = lb$.

2. In all *Triangles, Trapeza's, Poligons*, or other Irregular Figure, divided into *Triangles*, put the Baſe $= b$, the Perpendicular $= p$, and Area $= x$: Then $x = \dfrac{bp}{2}$.

Or put the 3 ſides of any *Triangle*, (*i. e.*) Baſe $= b$, greater ſide $= a$, leſſer $= l$, and Area $= x$: Then $x = \tfrac{1}{2} b \sqrt{ll - \overline{\tfrac{1}{2} b + \dfrac{ll - aa}{2b}}\Big|^{\frac{1}{2}}}$

3. In all *Circular Figures*, put the Diameter $= d$, Perifery $= p$ and Area $= x$: Then $x = \frac{dp}{4}$.

4. In a *Semicircle* or *Sector of a Circle*, put the Radius $= r$, Arch $= a$, and Area $= x$: Then $x = \frac{ra}{2}$.

5. In a *Segment of a Circle*, put $\frac{1}{2}$ the Arch $= a$, the Right Sine $= s$, Verſed Sine $= v$, and Area $= x$; alſo put $a + s = p$, and $a - s = q$: Then $x = \frac{qss + pvv}{2v}$.

6. In a *Circle* put $d =$ Diameter, $p =$ Perifery, $x =$ Area, as before : Then,
as 1 . 3,141592, &c. :: $d . p$, or as 7 to 22.
1 . 0,31831, &c. :: $p . d$, or as 22 to 7.
1 . 0,78540, &c. :: $dd . x$, or as 14 to 11.
1 . 1,27324, &c. :: $x . dd$, or as 11 to 14.
1 . 0,70711, &c. :: d : to ſide of a Squ. inſcrib'd
1 . 0,88623, &c. :: d . to ſide of a Squ. equal.

7. In *Eliptical Figures*, for the Tranſverſe Diam. put t, and the Conjugate $= c$, Area $= x$: Then $\sqrt{ct} =$ Diam. of a Circle of $=$ Area. $\therefore x = \frac{p\sqrt{ct}}{4}$.

8. In all *Parabolical Figures*, put the Abſiſſa $= a$, Ordinate $= o$, and the Area $= x$: Then $x = \frac{2ao}{3}$.

All Area's are to one another as the Squares of their like Sides.

9. In the Surface of a *Piramid*, put the Altitude $= a$, the Baſe of one ſide $= b$, the Number of ſuch ſides $= n$, and the Area $= x$: Then $\frac{abn}{2} = x$.

10. In the Surface of a *Cone*, put the Altitude of the Cone $= a$; Circumference of the Baſe $= b$, Area $= x$: Then $x = \frac{ab}{2}$.

11. In the Surface of a *Cylender* or *Priſme*, put the Length $= l$, Circumference $= c$, Area $= x$: Then $x = lc$.

12. In the Surface of a *Sphere*, put its Diameter $= d$, Circumference $= c$, and Area $= x$: Then $x = dc$.

III. *Of Solid Meaſure.*

$$\text{As}\ \begin{array}{l} 1 \,.\, 3 :: \textit{Cone} \,.\, \textit{Cylinder}, \\ 1 \,.\, 3 :: \textit{Piramid.}\ \textit{Priſme}, \\ 2 \,.\, 3 :: \textit{Sphere} \,.\, \textit{Cylinder}, \end{array} \left.\vphantom{\begin{array}{l}1\\1\\2\end{array}}\right\} \textit{\& vice verſa.}$$

1. In all *Cubes*, *Parallelapipids*, &c. put $l =$ Length, $b =$ Breadth, $t =$ Thickneſs, and $x =$ Solidity: Then $x = lbt$.

2. In all *Priſms*, *Cylenders*, &c. put Area Baſe $= a$, Length $= l$, $x =$ Solidity: Then $x = al$.

3. In all *Piramids* and *Cones*, &c. $x = \frac{al}{3}$.

4. In a *Cylender*, put d = Diameter, c = Circumference, Length = l, Solidity = x.

$$\text{As } 1 \,.\, 0{,}785399 :: ddl \,.\, x$$
$$1 : 0{,}079577 :: ccl \,.\, x.$$

5. In *Spheres* or *Globes*, put the Superficies = s, Diameter = d, Circumference = c, Solidity = x: Then $x = \frac{ds}{6}$.

$$\text{Or as } 1 \,.\, 0{,}523599 :: ddd \,.\, x.$$
$$1 \,.\, 0{,}016887 :: ccc \,.\, x.$$
$$1 \,.\, 1{,}90860 :: x \,.\, ddd.$$
$$1 \,.\, 0{,}80604 :: d \,.\, \surd x.$$
$$1 \,.\, 0{,}25655 :: c \,.\, \surd x.$$

6. In all *Spheroids*, put the Area of the leſſer Circle = a, longeſt Diameter = d: Then $x = \frac{2da}{3}$.

7. In all *Fruſtrums of Cones* or *Cylenders*, put the Greater Baſe = B; Leſſer Baſe = b, Length or Height = l, Solidity = x:

$$\text{Then } x = \tfrac{1}{3} l \times \overline{B + b + \sqrt{Bb}}.$$

8. In all *Segments of Spheres*, put the Area of Segments Baſe = r, Altitude = a, Solidity = x: Then $x = \frac{9ar}{16}$.

All *Solids* are to one another as the *Cubes* of their like ſides.

FINIS.